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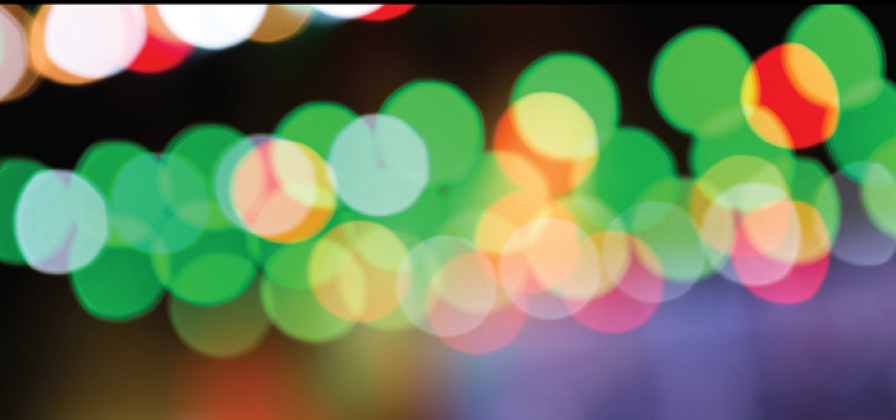
6TH EDITION

college algebra

graphs and models



College Algebra

Graphs and Models

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6TH EDITION

College Algebra

Graphs and Models

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Contents

Preface

College Algebra: Graphs and Models is known for enabling students to "see the math" through its

- focus on visualization,
- early introduction of functions,
- integration of technology, and
- connections between math concepts and the real world.

New!

With the new edition, we continue to innovate by positioning the review material as a more effective tool for teachers and students. Chapter R from the previous edition has been condensed into **28 Just-in-Time review topics** that are placed before Chapter 1. This new review feature is designed to give each student the opportunity to be successful in this course by providing a quick review of topics from intermediate algebra that will be built upon in new college algebra topics. The review can be used in an individualized instruction format since some students will require more review than others. Treating the review in this manner will allow more time to cover the college algebra topics in the syllabus.

On the other hand, some instructors might choose to review some or all of the topics with the entire class at the beginning of the course or in a just-in-time format as each is needed. We think that instructors will appreciate the flexibility that the Just-in-Time feature offers.

Additional resources in the MyMathLab course reflect the themes of just-in-time review and concept retention. For example, new Cumulative Review assignments allow students to synthesize and retain concepts learned throughout the course. The Just-in-Time review topics within MyMathLab allow for assignable Getting Ready review quizzes that lead to personalized Getting Ready homework focused on areas in which students need additional practice.

Our overarching goal is to provide students with a learning experience that will not only lead to success in this course, but also prepare them to be successful in the mathematics courses that they take in the future.

Content Changes to the Sixth Edition

New!

- **Just-in-Time Review** Review of prerequisite algebra topics is now presented when students need it most.
  - A set of 28 numbered, short review topics creates an efficient review of intermediate algebra topics:

1. Real Numbers
2. Properties of Real Numbers
3. Order on the Number Line
4. Absolute Value
5. Operations with Real Numbers
6. Interval Notation
7. Integers as Exponents
8. Scientific Notation
9. Order of Operations
10. Introduction to Polynomials
11. Add and Subtract Polynomials
12. Multiply Polynomials
13. Special Products of Binomials
14. Factor Polynomials; The FOIL Method

15. Factor Polynomials: The *ac*-Method
16. Special Factorizations
17. Equation-Solving Principles
18. Inequality-Solving Principles
19. The Principle of Zero Products
20. The Principle of Square Roots
21. Simplify Rational Expressions
22. Multiply and Divide Rational Expressions
23. Add and Subtract Rational Expressions
24. Simplify Complex Rational Expressions
25. Simplify Radical Expressions
26. Rationalize Denominators
27. Rational Exponents
28. The Pythagorean Theorem

Just in Time 10

- This feature is placed before Chapter 1. Just-in-Time icons are positioned throughout the text next to the appropriate example where review of an intermediate algebra topic would be helpful.

- **Cumulative Reviews** For enhanced concept review, cumulative reviews, assignable in MyMathLab, allow students to synthesize and retain concepts learned throughout the course.
- **Informed Exercises** We have analyzed the MyMathLab usage data which has informed our revision of the exercises for this new edition. The goal is to ultimately improve the quality and quantity of exercises that are most relevant.

Emphasis on Functions

Functions are the core of this course and are presented as a thread that runs throughout the course rather than as an isolated topic. We introduce functions in Chapter 1, whereas many traditional college algebra textbooks cover equation-solving in Chapter 1. Our approach of introducing students to a relatively new concept at the beginning of the course, rather than requiring them to begin with a review of material that was previously covered in intermediate algebra, immediately engages them and serves to help them avoid the temptation to neglect studying early in the course because "I already know this."

The concept of a function can be challenging for students. By repeatedly exposing them to the language, notation, and use of functions, demonstrating visually how functions relate to equations and graphs, and also showing how functions can be used to model real data, we hope to ensure that students not only become comfortable with functions but also come to understand and appreciate them. You will see this emphasis on functions woven throughout the other themes that follow.

Classify the Function Exercises With a focus on conceptual understanding, students are asked periodically to identify a number of functions by their type (linear, quadratic, rational, and so on). As students progress through the text, the variety of functions with which they are familiar increases and these exercises become more challenging. The "classifying the function" exercises appear with the review exercises in the Skill Maintenance portion of an exercise set. (See pp. 262 and 353–354.)

Visual Emphasis

Our early introduction of functions allows graphs to be used to provide a visual aspect to solving equations and inequalities. For example, we are able to show students both algebraically and visually that the solutions of a quadratic equation $ax^2 + bx + c = 0$ are the zeros of the quadratic function $f(x) = ax^2 + bx + c$, as well as the first coordinates of the x-intercepts of the graph of that function. This makes it possible for students, particularly visual learners, to gain a quick understanding of these concepts. (See pp. 178, 181, 221, 281, and 342.)

Visualizing the Graph Appearing at least once in every chapter, this feature provides students with an opportunity to match an equation with its graph by focusing on the characteristics of the equation and the corresponding attributes of the graph. (See pp. 138, 194, and 276.) In MyMathLab, animated Visualizing the Graph features for each chapter allow students to interact with graphs on an entirely new level. In addition to this full-page feature, many of the exercise sets include exercises in which the student is asked to match an equation with its graph or to find an equation of a function from its graph. (See pp. 140, 141, 230, and 326.)

Side-by-Side Examples Many examples are presented in a side-by-side, two-column format in which the algebraic solution of an equation appears in the left column and a graphical solution appears in the right column. (See pp. 244, 284, and 357.) This enables students to visualize and comprehend the connections among the solutions of an equation, the zeros of a function, and the *x*-intercepts of the graph of a function.

New!

Guided Visualizations These new figures help bring mathematical concepts to life. They are included in MyMathLab as both a teaching and a learning tool. Used as a lecture tool, the figures help engage students more fully and save the time that would otherwise be spent drawing figures by hand. Questions pertaining to each guided visualization are assignable in MyMathLab and reinforce active learning, critical thinking, and conceptual learning.

Integrated Technology In order to increase students' understanding of the course content through a visual means, we integrate graphing calculator technology throughout. The use of the graphing calculator is woven throughout the text's exposition, exercise sets, and testing program without sacrificing algebraic skills. Graphing calculator technology is included in order to enhance—not replace—students' mathematical skills, and to alleviate the tedium associated with certain procedures. (See pp. 176, 273–274, and 355.) The graphing calculator windows enhance the visual element of the text, providing graphical interpretations of solutions of equations, zeros of functions, and *x*-intercepts of graphs of functions.

Making Connections

Zeros, Solutions, and *x*-Intercepts We find that when students understand the connections among the real zeros of a function, the solutions of its associated equation, and the first coordinates of the *x*-intercepts of its graph, a door opens to a new level of mathematical comprehension that increases the probability of success in this course. We emphasize zeros, solutions, and *x*-intercepts throughout the text by using consistent, precise terminology and including exceptional graphics. Seeing this theme repeated in different contexts leads to a better understanding and retention of these concepts. (See pp. 171 and 181.)

Connecting the Concepts This feature highlights the importance of connecting concepts. When students are presented with concepts in visual form—using graphs, an outline, or a chart—rather than merely in paragraphs of text, comprehension is streamlined and retention is enhanced. The visual aspect of this feature invites students to stop and check their understanding of how concepts work together in one section or in several sections. This check in turn enhances student performance on homework assignments and exams. (See pp. 69, 181, and 249.)

Annotated Examples We have included over 730 annotated examples designed to fully prepare the student to work the exercises. Learning is carefully guided with the use of numerous color-coded art pieces and step-by-step annotations. Substitutions and annotations are highlighted in red for emphasis. (See pp. 175 and 349–350.)

Now Try Exercises Now Try Exercises are found after nearly every example. This feature encourages active learning by asking students to do an exercise in the exercise set that is similar to the example that the student has just read. (See pp. 173, 268, and 322.)

Synthesis Exercises These exercises appear at the end of each exercise set and encourage critical thinking by requiring students to synthesize concepts from several sections or to take a concept a step further than in the general exercises. For the Sixth Edition, these exercises are assignable in MyMathLab. (See pp. 251–252, 330, and 380.)

Real-Data Applications We encourage students to see and interpret the mathematics that appears every day in the world around them. Throughout the writing process, we conducted an energetic search for real-data applications, and the result is a variety of examples and exercises that connect the mathematical content with everyday life. Most of these applications feature source lines and many include charts and graphs. Many are drawn from the fields of health, business and economics, life and physical sciences, social science, and areas of general interest such as sports and travel. (See pp. 37 ("Food Stamp Program"), 63 ("Industrial Robots"), 141 ("Medical Care Abroad"), 184 ("Funding for Afghan Security"), 231 ("Vinyl Album Sales"), 328 ("Alfalfa Imported by China"), 405 ("Cosmetic Surgery"), 414 ("Top Art Auction Sales"), 491 ("The Ellipse at the White House"), and 545 ("The Economic Multiplier; Super Bowl XLVII").)

Ongoing Review

The most significant change to the Sixth Edition is the new Just-in-Time Review feature, designed to provide students with efficient and effective review of basic algebra skills.

New! Just-in-Time Review Chapter R has been condensed into 28 numbered, short review topics to create an efficient review of intermediate algebra topics. This feature is placed before Chapter 1.

New!

Just in Time 20

- Just-In-Time icons are placed throughout the text next to the example where review of an intermediate algebra topic would be helpful. (See pp. 33, 95, 164, 220, 315, and 396.)
- The coverage of each topic contains worked-out examples and a short exercise set. Answers to all exercises appear at the end of the answers at the back of the book.
- Worked-out solutions to all Just-in-Time exercises are included in the *Student Solutions Manual*.

Just in Time 20

THE PRINCIPLE OF SQUARE ROOTS

The principle of square roots can be used to solve some quadratic equations.

THE PRINCIPLE OF SQUARE ROOTS

If $x^2 = k$, then $x = -\sqrt{k}$ *or* $x = \sqrt{k}$.

EXAMPLES Solve.

1. $s^2 - 144 = 0$

$$s^2 = 144$$

$$s = -\sqrt{144} \quad or \quad s = \sqrt{144}$$

$$s = -12 \quad or \quad s = 12$$

The solutions are -12 and 12, or ± 12.

2. $3x^2 - 21 = 0$

$$3x^2 = 21$$

$$x^2 = 7$$

$$x = -\sqrt{7} \quad or \quad x = \sqrt{7}$$

The solutions are $-\sqrt{7}$ and $\sqrt{7}$, or $\pm\sqrt{7}$.

Do Exercises 1–6.

Solve.

1. $x^2 - 36 = 0$

2. $2y^2 - 20 = 0$

3. $6z^2 = 18$

4. $3t^2 - 15 = 0$

5. $z^2 - 1 = 24$

6. $5x^2 - 75 = 0$

Mid-Chapter Mixed Review This review reinforces understanding of the mathematical concepts and skills covered in the first half of the chapter before students move on to new material in the second half of the chapter. Each review begins with at least three true/false exercises that require students to consider the concepts they have studied and also contains exercises that drill the skills from all prior sections of the chapter. They are available as assignments in MyMathLab. (See pp. 121–122 and 346–347.)

Collaborative Discussion and Writing Exercises appear in the Mid-Chapter Mixed Review as well. These exercises can be discussed in small groups or by the class as a whole to encourage students to talk about the key mathematical concepts in the chapter. They can also be assigned to individual students to give them an opportunity to write about mathematics. (See pp. 199 and 253).

A section reference (shown in red) is provided for each exercise in the Mid-Chapter Mixed Review. This tells the student which section to refer to if help is needed to work the exercise. Answers to all exercises in the Mid-Chapter Mixed Review are given at the back of the book.

Study Guide This feature is found at the beginning of the **Summary and Review** near the end of each chapter. Presented in a two-column format and organized by section, this feature gives key concepts and terms in the left column and a worked-out example in the right column. It provides students with a concise and effective review of the chapter that is a solid basis for studying for a test. In MyMathLab, these Study Guides are accompanied by narrated examples to reinforce the key concepts and ideas. (See pp. 210–215 and 381–387.)

Exercise Sets There are over 5040 exercises in this text. The exercise sets are enhanced with real-data applications and source lines, detailed art pieces, tables, graphs, and photographs. In addition to the exercises that provide students with concepts presented in the section, the exercise sets feature the following elements to provide ongoing review of topics presented earlier:

- **Skill Maintenance Exercises.** These exercises provide an ongoing review of concepts previously presented in the course, enhancing students' retention of these concepts. They include **Vocabulary Reinforcement**, described next, and **Classifying the Function** exercises, described earlier in the section "Emphasis on Functions." A section reference (shown in red) is provided for each exercise. This tells the student which section to refer to if help is needed to work the exercise. Answers to all Skill Maintenance exercises appear in the answer section at the back of the book. (See pp. 128, 206, 279–280, and 345.)
- **Enhanced Vocabulary Reinforcement Exercises.** This feature checks and reviews students' understanding of the vocabulary introduced throughout the text. It appears once in every chapter, in the Skill Maintenance portion of an exercise set, and is intended to provide a continuing review of the terms that students must know in order to be able to communicate effectively in the language of mathematics. (See pp. 149–150, 209, and 279.)
- **Enhanced Synthesis Exercises.** These exercises appear at the end of each exercise set and encourage critical thinking by requiring students to synthesize concepts from several sections or to take a concept a step further than in the general exercises. For the Sixth Edition, these exercises are assignable in MyMathLab.

Review Exercises These exercises in the **Summary and Review** supplement the Study Guide by providing a thorough and comprehensive review of the skills taught in the chapter. A group of true/false exercises appears first, followed by a large number of exercises that drill the skills and concepts taught in the chapter. In addition, three multiple-choice exercises, one of which involves identifying the graph of a function, are included in the Review Exercises for every chapter. Each review exercise is accompanied by a section reference that, as in the Mid-Chapter Mixed Review, directs students to the section in which the material being reviewed can be found. Also included are Collaborative Discussion and Writing exercises are also included. These exercises are described under the Mid-Chapter Mixed Review heading on p. xv. (See pp. 215–217 and 388–391.)

Chapter Test The test at the end of each chapter allows students to test themselves and target areas that need further study before taking the in-class test. Each Chapter Test includes a multiple-choice exercise involving identifying the graph of a function. Answers to all questions in the Chapter Tests appear in the answer section at the back of the book, along with corresponding section references. (See pp. 218 and 391–392.)

Review Icons Placed next to the concept that a student is currently studying, a review icon references a section of the text in which the student can find and review topics on which the current concept is built. (See pp. 263 and 308.)

DOMAIN
REVIEW SECTION 1.2.

Acknowledgments

We wish to express our heartfelt thanks to a number of people who have contributed in special ways to the development of this textbook. Our editor, Chelsea Kharakozova, and Editor in Chief, Anne Kelly, encouraged and supported our vision. We are very appreciative of the marketing insight provided by Peggy Lucas and Rachel Ross, our marketing managers, and of the support that we received from the entire Pearson team, including Rachel Reeve, project manager, Barbara Atkinson, cover designer, Ashley Gordon, editorial assistant, and Fiona Murray, marketing assistant. We also thank Erica Lange, media producer, for her creative work on the media products that accompany this text. And we are immensely grateful to Martha Morong for her editorial and production services, and to Geri Davis for her text design and art editing, and for the endless hours of hard work they have done to make this a book of which we are proud. We also thank Laurie Hurley and Holly Martinez for their meticulous accuracy checking and proofreading of the text.

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* *Reviewers of the Sixth Edition*

M.L.B.
J.A.B.
D.J.E.
J.A.P.

BREAKTHROUGH
To improving results

Resources for Success

MyMathLab® Online Course for *College Algebra: Graphs and Models* by Bittinger/Beecher/Ellenbogen/Penna

(access code required)

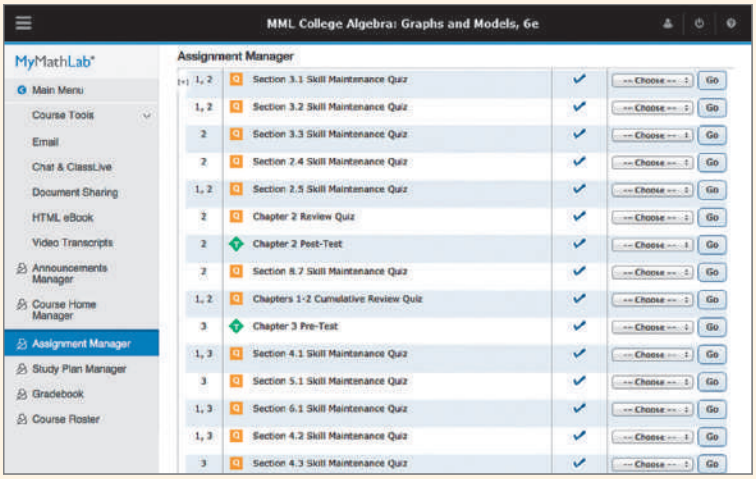
MyMathLab is available to accompany Pearson's market-leading text offerings. To give students a consistent tone, voice, and teaching method, each text's flavor and approach is tightly integrated throughout the accompanying MyMathLab course, making learning the material as seamless as possible.

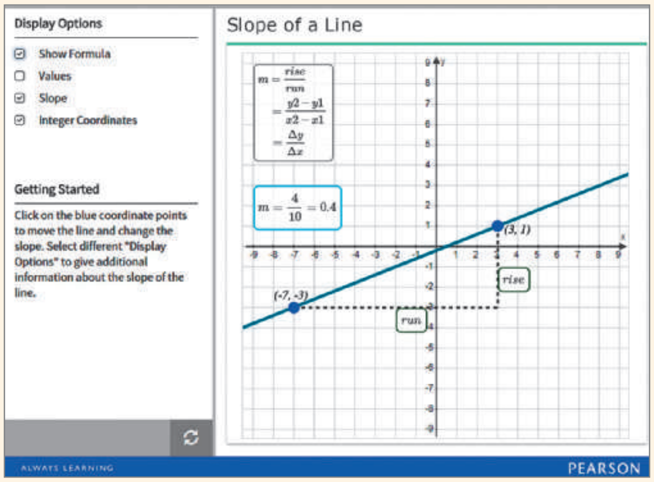
With this new edition, instructors will find even more assignable exercises in MyMathLab to match many of the in-text features so many instructors have grown to rely on, as well as a new Skills for Success Module.

Skill Maintenance Quizzes and Cumulative Reviews

Instructors can now assign MyMathLab quizzes generated from the *Skill Maintenance Exercises* found in the text. These quizzes support ongoing review to help students maintain essential skills.

For enhanced concept review throughout the course, *Cumulative Reviews* are now also assignable in MyMathLab. These assignments allow students to synthesize and retain concepts.





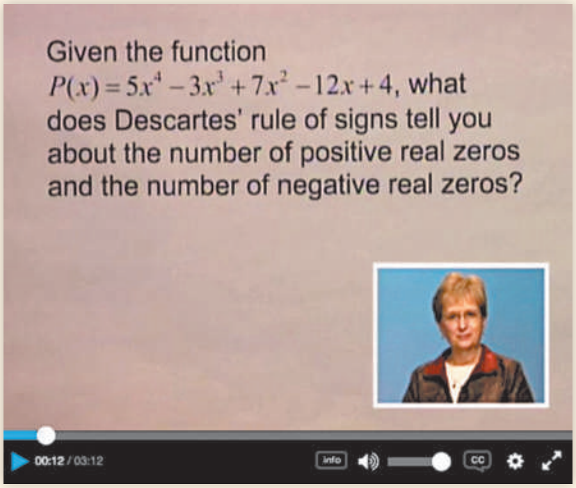
NEW! Guided Interactive Figures

These engaging interactive figures bring mathematical concepts to life, helping students visualize the concepts through directed explorations and purposeful manipulation. Guided Visualizations are assignable in MyMathLab and encourage active learning, critical thinking, and conceptual learning.

www.mymathlab.com

Skills for Success Modules

Skills for Success Modules help foster success in collegiate courses and prepare students for future professions. Topics such as "Time Management," "Stress Management" and "Financial Literacy" are available within the MyMathLab course. Instructors can integrate these media-rich activities with their traditional MyMathLab assignments.



Video Assessment Exercises

Featuring authors Judy Beecher and Judy Penna, these Example Solutions videos walk students through the detailed solution process for nearly all examples from the text. Corresponding exercises check for conceptual understanding. Videos and exercises are assignable in MyMathLab.

Instructor Resources

Additional resources can be downloaded from within MyMathLab and at www.pearsonhighered.com, or hardcopy resources can be ordered from your Pearson sales representative.

Annotated Instructor's Edition

The instructor's edition includes all answers to the exercise sets. Shorter answers are presented on the same page as the exercise; longer answers are in the back of the text. Sample homework assignments are indicated by a blue underline and may be assigned in MyMathLab®. Available upon request from your Pearson sales representative.

Instructor's Solutions Manual (Download Only)

Written by author Judy Penna, this resource contains worked-out solutions to all exercises in the exercise sets, Mid-Chapter Mixed Reviews, Chapter Reviews, and Chapter Tests, as well as solutions for all the Just-In-Time exercises.

PowerPoint® Lecture Slides

Feature presentations written and designed specifically for this text. These lecture slides provide an outline for presenting definitions, figures, and key examples from the text.

Online Test Bank (Download Only)

Contains four free-response tests forms for each chapter following the same format and having the same level of difficulty as the test in the main text and two multiple-choice test forms for each chapter. It also provides six forms of the final examination, four with free-response questions and two with multiple-choice questions.

TestGen®

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

Learning Catalytics Integration

MyMathLab now provides **Learning Catalytics**, an interactive student response tool that uses students' smartphones, tablets, or laptops to engage them in more sophisticated tasks and thinking. Learning Catalytics contains Pearson-created content for College Algebra that allows instructors to take advantage of this exciting technology immediately.

Student Resources

Additional resources to promote student success.

Author Example Videos

Ideal for distance learning or supplemental instruction, these video feature authors Judy Beecher and Judy Penna working through and explaining examples in the text. Assignable in MyMathLab with new Video Assessment exercises.

Video Notebook

This notebook can accompany the text and/or MyMathLab course. It contains fill-in-the-blank worksheets to accompany the video examples presented by the authors. Key definitions, theorems, and procedures are also included. After filling in the worksheet while watching the video, the student has an excellent study guide for review and test preparation. This is available electronically for download within MyMathLab or as a printed resource.

Student's Solutions Manual

Written by author Judy Penna, this resource contains completely worked-out solutions with step-by-step annotations for all the odd-numbered exercises in the exercise sets, Mid-Chapter Mixed Reviews, and Chapter Reviews, as well as solutions for all the Chapter Test exercises and the Just-In-Time exercises.

Chapter R: Basic Concepts of Algebra

Available within MyMathLab, Chapter R supplements the prerequisite topics of the Just-in-Time review with more in-depth coverage and exercises.

R.1 The Real-Number System
R.2 Integer Exponents, Scientific Notation, and Order of Operations
R.3 Addition, Subtraction, and Multiplication of Polynomials
R.4 Factoring
R.5 The Basics of Equation Solving
R.6 Rational Expressions
R.7 Radical Notation and Rational Exponents

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To the Student

Guide to Success

Success can be planned. Combine goals and good study habits to create a plan for success that works for you. The following list contains study tips that your authors consider most helpful.

Skills for Success

- **Set goals and expect success.** Approach this class experience with a positive attitude.
- **Communicate with your instructor** when you need extra help.
- **Take your text with you to class and lab.** Each section in the text is designed with headings and boxed information that provide an outline for easy reference.
- **Ask questions in class, lab, and tutoring sessions.** Instructors encourage them, and other students probably have the same questions.
- **Begin each homework assignment as soon as possible.** If you have difficulty, you will then have the time to access supplementary resources.
- **Carefully read the instructions** before working homework exercises **and include all steps**.
- **Form a study group** with fellow students. Verbalizing questions about topics that you do not understand can clarify the material for you.
- After each quiz or test, **write out corrected step-by step solutions** to all missed questions. They will provide a valuable study guide for the midterm exam and the final exam.
- **MyMathLab has numerous tools to help you succeed.** Use MyMathLab to create a personalized study plan and practice skills with sample quizzes and tests.
- **Knowing math vocabulary is an important step toward success.** Review vocabulary using the Vocabulary Reinforcement exercises in the text and in MyMathLab.
- If you miss a lecture, **watch the video in the Multimedia Library** of MyMath Lab that explains the concepts you missed.

In writing this textbook, we challenged ourselves to do everything possible to help you learn the concepts and skills contained between its covers so that you will be successful in this course and in the mathematics courses you take in the future. We realize that your time is both valuable and limited, so we communicate in a highly visual way that allows you to learn quickly and efficiently. We are confident that, if you invest an adequate amount of time in the learning process, this text will be of great value to you. We wish you a positive learning experience.

Marv Bittinger
Judy Beecher
David Ellenbogen
Judy Penna

JUST-IN-TIME Review

Throughout this text, there are Just-in-Time icons, numbered 1–28, that refer to brief reviews of the following 28 intermediate-algebra topics. Each mini-review lesson is accompanied by several exercises. All answers are provided in the answer section at the back of the text.

1. Real Numbers
2. Properties of Real Numbers
3. Order on the Number Line
4. Absolute Value
5. Operations with Real Numbers
6. Interval Notation
7. Integers as Exponents
8. Scientific Notation
9. Order of Operations
10. Introduction to Polynomials
11. Add and Subtract Polynomials
12. Multiply Polynomials
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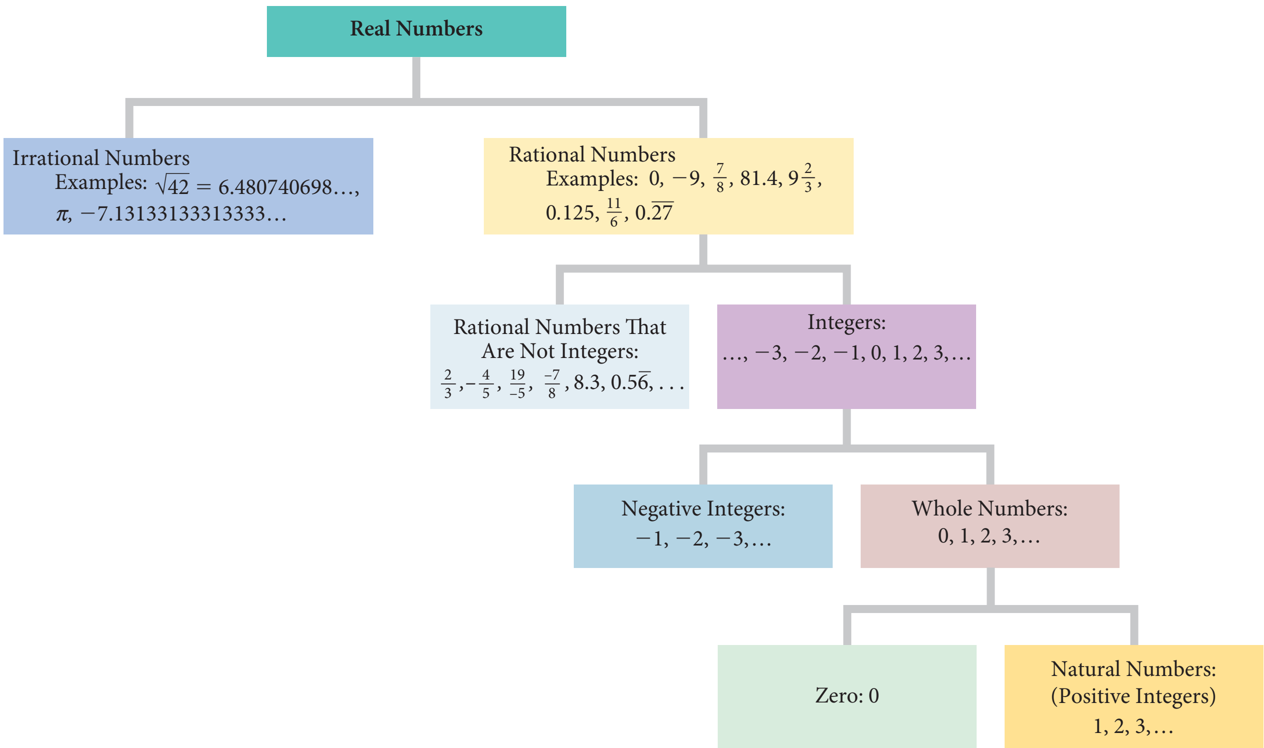
14. Factor Polynomials; The FOIL Method
15. Factor Polynomials: The *ac*-Method
16. Special Factorizations
17. Equation-Solving Principles
18. Inequality-Solving Principles
19. The Principle of Zero Products
20. The Principle of Square Roots
21. Simplify Rational Expressions
22. Multiply and Divide Rational Expressions
23. Add and Subtract Rational Expressions
24. Simplify Complex Rational Expressions
25. Simplify Radical Expressions
26. Rationalize Denominators
27. Rational Exponents
28. The Pythagorean Theorem

Just in Time 1

REAL NUMBERS

Some frequently used sets of real numbers and the relationships among them are shown below.



(continued)

Numbers that can be expressed in the form p/q, where p and q are integers and $q \neq 0$, are **rational numbers**. Decimal notation for rational numbers either *terminates* (ends) or *repeats*. Each of the following is a rational number:

$$0, \quad -17, \quad \frac{13}{4}, \quad \sqrt{25} = 5;$$

$$\frac{1}{4} = 0.25 \quad \text{(terminating decimal);}$$

$$-\frac{5}{11} = -0.454545\ldots = -0.\overline{45} \quad \text{(repeating decimal);}$$

$$\frac{5}{6} = 0.8333\ldots = 0.8\overline{3} \quad \text{(repeating decimal).}$$

The real numbers that are not rational are **irrational numbers**. Decimal notation for irrational numbers neither terminates nor repeats. Each of the following is an irrational number. Note in each case that there is no repeating block of digits.

$\sqrt{2} = 1.414213562\ldots,$

$\sqrt{65} = 8.062257748\ldots,$

$-6.12122122212222\ldots,$

$\pi = 3.1415926535\ldots$

($\frac{22}{7}$ and 3.14 are *rational approximations* of the irrational number π.)

The set of all rational numbers combined with the set of all irrational numbers gives us the set of **real numbers.**

Do Exercises 1–6.

In Exercises 1–6, consider the numbers

$\frac{2}{3}$, 6, $\sqrt{3}$, -2.45, $\sqrt[6]{26}$, $18.\overline{4}$, -11, $\sqrt[3]{27}$, $5\frac{1}{6}$, $7.151551555\ldots$, $-\sqrt{35}$, $\sqrt[5]{3}$, $-\frac{8}{7}$, 0, $\sqrt{16}$.

1. Which are rational numbers?

2. Which are rational numbers but not integers?

3. Which are irrational numbers?

4. Which are integers?

5. Which are whole numbers?

6. Which are real numbers?

All answers to the Just-in-Time exercises appear at the end of the answers at the back of the book.

Just in Time 2

PROPERTIES OF REAL NUMBERS

PROPERTIES OF REAL NUMBERS

For any real numbers a, b, and c:

$a + b = b + a$ and $ab = ba$	Commutative properties of addition and multiplication
$a + (b + c) = (a + b) + c$ and $a(bc) = (ab)c$	Associative properties of addition and multiplication
$a + 0 = 0 + a = a$	Additive identity property
$-a + a = a + (-a) = 0$	Additive inverse property
$a \cdot 1 = 1 \cdot a = a$	Multiplicative identity property
$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \ (a \neq 0)$	Multiplicative inverse property
$a(b + c) = ab + ac$ and $a(b - c) = ab - ac$	Distributive property

EXAMPLES Name the property illustrated in each sentence.

1. $8 \cdot 5 = 5 \cdot 8$	Commutative property of multiplication
2. $14 + (-14) = 0$	Additive inverse property
3. $2(a - b) = 2a - 2b$	Distributive property
4. $5 + (m + n) = (5 + m) + n$	Associative property of addition
5. $6 \cdot 1 = 1 \cdot 6 = 6$	Multiplicative identity property
6. $q + t = t + q$	Commutative property of addition
7. $\frac{7}{9} + 0 = \frac{7}{9}$	Additive identity property
8. $-2(cd) = (-2c)d$	Associative property of multiplication
9. $3(x + y) = 3x + 3y$	Distributive property
10. $8 \cdot \frac{1}{8} = 1$	Multiplicative inverse property

Do Exercises 1–10.

Name the property illustrated by the sentence.

1. $-24 + 24 = 0$

2. $7(xy) = (7x)y$

3. $9(r - s) = 9r - 9s$

4. $11 + z = z + 11$

5. $-20 \cdot 1 = -20$

6. $5(x + y) = (x + y)5$

7. $q + 0 = q$

8. $75 \cdot \frac{1}{75} = 1$

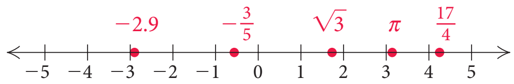
9. $(x + y) + w = x + (y + w)$

10. $8(a + b) = 8a + 8b$

All answers to the Just-in-Time exercises appear at the end of the answers at the back of the book.

Just in Time 3 ORDER ON THE NUMBER LINE

The real numbers are modeled using a **number line,** as shown below. Each point on the line represents a real number, and every real number is represented by a point on the line.



The order of the real numbers can be determined from the number line. If a number a is to the left of a number b, then a **is less than** b $(a < b)$. Similarly, a **is greater than** b $(a > b)$ if a is to the right of b on the number line. For example, we see from the number line above that $-2.9 < -\frac{3}{5}$, because -2.9 is to the left of $-\frac{3}{5}$. Also, $\frac{17}{4} > \sqrt{3}$, because $\frac{17}{4}$ is to the right of $\sqrt{3}$.

The statement $a \leq b$, read "a is less than or equal to b," is true if either $a < b$ is true or $a = b$ is true. A similar statement holds for $a \geq b$.

Classify the inequality as true or false.

1. $9 < -9$
2. $-10 \leq -1$
3. $-\sqrt{26} < -5$
4. $\sqrt{6} \geq \sqrt{6}$
5. $-30 > -25$
6. $-\frac{4}{5} > -\frac{5}{4}$

Do Exercises 1–6.

Just in Time 4 ABSOLUTE VALUE

The **absolute value** of a number a, denoted $|a|$, is its distance from 0 on the number line. For example, $|-5| = 5$, because the distance of -5 from 0 is 5. For any real number a,

$$|a| = a \quad \text{if } a \geq 0 \qquad \text{and} \qquad |a| = -a \quad \text{if } a < 0.$$

EXAMPLES Simplify.

1. $|-10| = 10$ **2.** $|0| = 0$ **3.** $\left|\frac{4}{9}\right| = \frac{4}{9}$

Absolute value can be used to find the distance between two points on the number line. For any real numbers a and b, the distance between a and b is $|a - b|$ or, equivalently, $|b - a|$.

EXAMPLE 4 Find the distance between -2 and 3.

$$|-2 - 3| = |-5| = 5,$$

or equivalently,

$$|3 - (-2)| = |3 + 2| = |5| = 5$$

Simplify.

1. $|-98|$
2. $|0|$
3. $|4.7|$
4. $\left|-\frac{2}{3}\right|$

Find the distance between the given pair of points on the number line.

5. $-7,\ 13$
6. $2,\ 14.6$
7. $-39,\ -28$
8. $-\frac{3}{4},\ \frac{15}{8}$

Do Exercises 1–8.

Just in Time 5

OPERATIONS WITH REAL NUMBERS

RULES FOR OPERATIONS WITH REAL NUMBERS

Addition

- *Positive Numbers*: Add the same way that we add arithmetic numbers. The answer is positive.
- *Negative Numbers*: Add absolute values. The answer is negative.
- *A Positive Number and a Negative Number:* If the numbers have the same absolute value, the answer is 0. If the numbers have different absolute values, subtract the smaller absolute value from the larger. If the positive number has the greater absolute value, the answer is positive. If the negative number has the greater absolute value, the answer is negative.

Subtraction

- To subtract, add the opposite, or additive inverse, of the number being subtracted.

Multiplication and Division, where the divisor is nonzero

- Multiply or divide the absolute values. If the signs are the same, the answer is positive. If the signs are different, the answer is negative.

EXAMPLES Add.

1. $9 + (-29) = -20$
2. $-9 + (-29) = -38$
3. $-9 + 29 = 20$

EXAMPLES Subtract.

4. $15 - 6 = 15 + (-6) = 9$
5. $15 - (-6) = 15 + 6 = 21$
6. $-15 - 6 = -15 + (-6) = -21$
7. $-15 - (-6) = -15 + 6 = -9$

EXAMPLES Multiply or divide.

8. $-5 \cdot 20 = -100$
9. $32 \div (-4) = -8$
10. $-32 \div 4 = -8$
11. $-32 \div (-4) = 8$
12. $-5 \cdot (-20) = 100$
13. $5 \cdot 20 = 100$

Do Exercises 1–15.

Compute and simplify.

1. $8 - (-11)$
2. $-\frac{3}{10} \cdot \left(-\frac{1}{3}\right)$
3. $15 \div (-3)$
4. $-4 - (-1)$
5. $7 \cdot (-50)$
6. $-0.5 - 5$
7. $-3 + 27$
8. $-400 \div (-40)$
9. $4.2 \cdot (-3)$
10. $-13 - (-33)$
11. $-60 + 45$
12. $\frac{1}{2} - \frac{2}{3}$
13. $-24 \div 3$
14. $-6 + (-16)$
15. $-\frac{1}{2} \div \left(-\frac{5}{8}\right)$

Just in Time 6

INTERVAL NOTATION

Sets of real numbers can be expressed using **interval notation**. For example, for real numbers a and b such that $a < b$, the **open interval** (a, b) is the set of real numbers between, but not including, a and b.

Some intervals extend without bound in one or both directions. The interval $[a, \infty)$, for example, begins at a and extends to the right without bound. The bracket indicates that a is included in the interval.

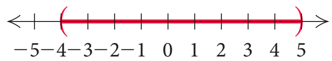
SET NOTATION	INTERVAL NOTATION	GRAPH
$\{x \mid a < x < b\}$	(a, b)	
$\{x \mid a \le x \le b\}$	$[a, b]$	
$\{x \mid a \le x < b\}$	$[a, b)$	
$\{x \mid a < x \le b\}$	$(a, b]$	
$\{x \mid x > a\}$	(a, ∞)	
$\{x \mid x \ge a\}$	$[a, \infty)$	
$\{x \mid x < b\}$	$(-\infty, b)$	
$\{x \mid x \le b\}$	$(-\infty, b]$	
$\{x \mid x \text{ is a real number}\}$	$(-\infty, \infty)$	

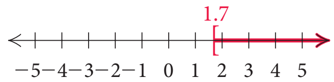
Do Exercises 1–10.

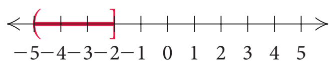
Write interval notation.

1. $\{x \mid -5 \le x \le 5\}$
2. $\{x \mid -3 < x \le -1\}$
3. $\{x \mid x \le -2\}$
4. $\{x \mid x > 3.8\}$
5. $\{x \mid 7 < x\}$
6. $\{x \mid -2 < x < 2\}$

Write interval notation for the graph.

7. 

8. 

9. 

10. 

Just in Time 7

INTEGERS AS EXPONENTS

When a positive integer is used as an *exponent*, it indicates the number of times that a factor appears in a product. For example, 7^3 means $7 \cdot 7 \cdot 7$, where 7 is the **base** and 3 is the **exponent**.

For any nonzero numbers a and b and any integers m and n,

$$a^0 = 1, \qquad a^{-m} = \frac{1}{a^m}, \quad \text{and} \quad \frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}.$$

PROPERTIES OF EXPONENTS

For any real numbers a and b and any integers m and n, assuming 0 is not raised to a nonpositive power:

$a^m \cdot a^n = a^{m+n}$	Product rule
$\frac{a^m}{a^n} = a^{m-n} \ (a \neq 0)$	Quotient rule
$(a^m)^n = a^{mn}$	Power rule
$(ab)^m = a^m b^m$	Raising a product to a power
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \ (b \neq 0)$	Raising a quotient to a power

EXAMPLES Simplify each of the following.

1. $4^2 \cdot 4^{-5} = 4^{2+(-5)} = 4^{-3}$, or $\frac{1}{4^3}$

2. $\left(\frac{7}{9}\right)^0 = 1$

3. $(8^2)^{-5} = 8^{2(-5)} = 8^{-10}$, or $\frac{1}{8^{10}}$

4. $\frac{x^{11}}{x^4} = x^{11-4} = x^7$

5. $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

6. $(cd)^{-2} = c^{-2}d^{-2}$, or $\frac{1}{c^2d^2}$

Do Exercises 1–10.

Simplify.

1. 3^{-6}

2. $\frac{1}{(0.2)^{-5}}$

3. $\frac{w^{-4}}{z^{-9}}$

4. $\left(\frac{z}{y}\right)^2$

5. 100^0

6. $\frac{a^5}{a^{-3}}$

7. $(2xy^3)(-3x^{-5}y)$

8. $x^{-4} \cdot x^{-7}$

9. $(mn)^{-6}$

10. $(t^{-5})^4$

Just in Time 8 — SCIENTIFIC NOTATION

We can use scientific notation to name both very large and very small positive numbers and to perform computations.

> **SCIENTIFIC NOTATION**
>
> **Scientific notation** for a number is an expression of the type
>
> $$N \times 10^m,$$
>
> where $1 \le N < 10$, N is in decimal notation, and m is an integer.

EXAMPLES

Convert to scientific notation.

1. 9,460,000,000,000

Since 9,460,000,000,000 is a large number, the exponent on 10 will be positive. We want to move the decimal point between 9 and 4. This is a move of 12 places, so the exponent will be 12.

$$9{,}460{,}000{,}000{,}000 = 9.46 \times 10^{12}$$

2. 0.000073

Since 0.000073 is a small number, the exponent on 10 will be negative. We want to move the decimal point between 7 and 3. This is a move of 5 places, so the exponent will be -5.

$$0.000073 = 7.3 \times 10^{-5}$$

Convert to decimal notation.

3. 5.4×10^7

The exponent is positive. We will move the decimal point 7 places to the right.

$$5.4 \times 10^7 = 54{,}000{,}000$$

4. 3.819×10^{-3}

The exponent is negative. We will move the decimal point 3 places to the left.

$$3.819 \times 10^{-3} = 0.003819$$

Do Exercises 1–8.

Convert to scientific notation.

1. 18,500,000

2. 0.000786

3. 0.0000000023

4. 8,927,000,000

Convert to decimal notation.

5. 4.3×10^{-8}

6. 5.17×10^6

7. 6.203×10^{11}

8. 2.94×10^{-5}

Just in Time 9

ORDER OF OPERATIONS

Recall that to simplify the expression $3 + 4 \cdot 5$, first we multiply 4 and 5 to get 20 and then we add 3 to get 23. Mathematicians have agreed on the following procedure, or rules for order of operations.

RULES FOR ORDER OF OPERATIONS

1. Do all calculations within grouping symbols before operations outside. When nested grouping symbols are present, work from the inside out.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in order from left to right.
4. Do all additions and subtractions in order from left to right.

EXAMPLES

1. $8(5-3)^3 - 20$
$= 8 \cdot 2^3 - 20$
$= 8 \cdot 8 - 20$
$= 64 - 20$
$= 44$

2. $10[7 - 4(8-5)]$
$= 10[7 - 4 \cdot 3]$
$= 10[7 - 12]$
$= 10[-5]$
$= -50$

3. $-32 \div 2 \times (-4) \div (-2)$
$= -16 \times (-4) \div (-2)$
$= 64 \div (-2)$
$= -32$

4. $\dfrac{10 \div (8-6) + 9 \cdot 4}{2^5 + 3^2}$
$= \dfrac{10 \div 2 + 9 \cdot 4}{32 + 9}$
$= \dfrac{5 + 36}{41}$
$= \dfrac{41}{41}$
$= 1$

5. $3^{12} \cdot 3^{-4} \div 3^3 \cdot 3^{-1}$
$= 3^8 \div 3^3 \cdot 3^{-1}$
$= 3^5 \cdot 3^{-1}$
$= 3^4$
$= 81$

Do Exercises 1–8.

Calculate.

1. $3 + 18 \div 6 - 3$
2. $5 \cdot 3 + 8 \cdot 3^2 + 4(6-2)$
3. $5(3 - 8 \cdot 3^2 + 4 \cdot 6 - 2)$
4. $16 \div 4 \cdot 4 \div 2 \cdot 256$
5. $2^6 \cdot 2^{-3} \div 2^{10} \div 2^{-8}$
6. $\dfrac{4(8-6)^2 - 4 \cdot 3 + 2 \cdot 8}{3^1 + 19^0}$
7. $64 \div [(-4) \div (-2)]$
8. $6[9 - (3-2)] + 4(2-3)$

Just in Time 10 — INTRODUCTION TO POLYNOMIALS

Polynomials are a type of algebraic expression that you will often encounter in your study of algebra. Some examples of polynomials are

$$3x - 4y, \quad 5y^3 - \tfrac{7}{3}y^2 + 3y - 2, \quad -2.3a^4,$$
$$16, \quad \text{and} \quad z^6 - \sqrt{5}.$$

Algebraic expressions like $8x - 13$, $x^2 + 3x - 4$, and $3a^5 - 11 + a$ are **polynomials in one variable.** Algebraic expressions like $3ab^3 - 8$ and $5x^4y^2 - 3x^3y^8 + 7xy^2 + 6$ are **polynomials in several variables.** The terms of a polynomial are separated by $+$ signs. The terms of $2x^2 - 8x + 3 = 2x^2 + (-8x) + 6$ are $2x^2$, $-8x$, and 3. The **degree of a term** is the sum of the exponents of the variables in that term. The **degree of a polynomial** is the degree of the term of highest degree.

A polynomial with just one term, like $-9y^6$, is a **monomial.** If a polynomial has two terms, like $x^2 + 4$, it is a **binomial.** A polynomial with three terms, like $4x^2 - 4xy + 1$, is a **trinomial.**

EXAMPLES Determine the degree of the polynomial.

1. $2x^3 - 1$ Degree: 3
2. $-5 \quad (-5 = -5x^0)$ Degree: 0
3. $w^2 - 3.5 + 4w^5 = 4w^5 + w^2 - 3.5$ Degree: 5
4. $7xy^3 - 16x^2y^4$ Degree: $2 + 4$, or 6

Do Exercises 1–8.

Determine the degree of the polynomial.

1. $5 - x^6$
2. $x^2y^5 - x^7y + 4$
3. $2a^4 - 3 + a^2$
4. -41
5. $4x - x^3 + 0.1x^8 - 2x^5$

Classify the polynomial as a monomial, a binomial, or a trinomial.

6. $x - 3$
7. $14y^5$
8. $2y - \dfrac{1}{4}y^2 + 8$

Just in Time 11 — ADD AND SUBTRACT POLYNOMIALS

If two terms of an expression have the same variables raised to the same powers, they are called **like terms**, or **similar terms**. We can **combine**, or **collect, like terms** using the distributive property. For example, $3y^2$ and $5y^2$ are like terms and $3y^2 + 5y^2 = (3 + 5)y^2 = 8y^2$. We add or subtract polynomials by combining like terms.

EXAMPLES Add or subtract each of the following.

1. $(-5x^3 + 3x^2 - x) + (12x^3 - 7x^2 + 3)$
$= (-5x^3 + 12x^3) + (3x^2 - 7x^2) - x + 3$
$= (-5 + 12)x^3 + (3 - 7)x^2 - x + 3$
$= 7x^3 - 4x^2 - x + 3$
2. $(6x^2y^3 - 9xy) - (5x^2y^3 - 4xy)$
$= 6x^2y^3 - 9xy - 5x^2y^3 + 4xy$
$= x^2y^3 - 5xy$

Do Exercises 1–5.

Add or subtract.

1. $(8y - 1) - (3 - y)$
2. $(3x^2 - 2x - x^3 + 2)$
$- (5x^2 - 8x - x^3 + 4)$
3. $(2x + 3y + z - 7)$
$+ (4x - 2y - z + 8)$
$+ (-3x + y - 2z - 4)$
4. $(3ab^2 + 4a^2b - 2ab + 6)$
$+ (-ab^2 - 5a^2b + 8ab + 4)$
5. $(5x^2 + 4xy - 3y^2 + 2)$
$- (9x^2 - 4xy + 2y^2 - 1)$

Just in Time 12

MULTIPLY POLYNOMIALS

To multiply monomials, we first multiply their coefficients, and then we multiply their variables.

EXAMPLES

1. $(-2x^3)(5x^4) = (-2 \cdot 5)(x^3 \cdot x^4) = -10x^7$
2. $(3yz^2)(8y^3z^5) = (3 \cdot 8)(y \cdot y^3)(z^2 \cdot z^5) = 24y^4z^7$

We can find the product of two binomials by multiplying the **F**irst terms, then the **O**uter terms, then the **I**nner terms, then the **L**ast terms. Then we combine like terms, if possible. This procedure is sometimes called **FOIL**.

EXAMPLE 3 Multiply: $(2x - 7)(3x + 4)$.

F L — F O I L

$$(2x - 7)(3x + 4) = 6x^2 + 8x - 21x - 28$$
$$= 6x^2 - 13x - 28$$

I O

EXAMPLE 4 Multiply: $(5c - d)(4c - 9d)$.

$$(5c - d)(4c - 9d) = 20c^2 - 45cd - 4cd + 9d^2$$
$$= 20c^2 - 49cd + 9d^2$$

Do Exercises 1–6.

Multiply.

1. $(3a^2)(-7a^4)$
2. $(y - 3)(y + 5)$
3. $(x + 6)(x + 3)$
4. $(2a + 3)(a + 5)$
5. $(2x + 3y)(2x + y)$
6. $(11t - 1)(3t + 4)$

Just in Time 13

SPECIAL PRODUCTS OF BINOMIALS

SPECIAL PRODUCTS OF BINOMIALS

$(A + B)^2 = A^2 + 2AB + B^2$	Square of a sum
$(A - B)^2 = A^2 - 2AB + B^2$	Square of a difference
$(A + B)(A - B) = A^2 - B^2$	Product of a sum and a difference

EXAMPLES

1. $(4x + 1)^2 = (4x)^2 + 2 \cdot 4x \cdot 1 + 1^2$
   $= 16x^2 + 8x + 1$
2. $(3y^2 - 2)^2 = (3y^2)^2 - 2 \cdot 3y^2 \cdot 2 + 2^2$
   $= 9y^4 - 12y^2 + 4$
3. $(x^2 + 3y)(x^2 - 3y) = (x^2)^2 - (3y)^2$
   $= x^4 - 9y^2$

Do Exercises 1–6.

Multiply.

1. $(x + 3)^2$
2. $(5x - 3)^2$
3. $(2x + 3y)^2$
4. $(a - 5b)^2$
5. $(n + 6)(n - 6)$
6. $(3y + 4)(3y - 4)$

Just in Time 14

FACTOR POLYNOMIALS; THE FOIL METHOD

When a polynomial is to be factored, we should always look first to factor out a factor that is common to all the terms using the distributive property. We generally look for the constant common factor with the largest absolute value and for variables with the largest exponent common to all the terms.

EXAMPLE 1 Factor: $15 + 10x - 5x^2$.

$$15 + 10x - 5x^2 = 5 \cdot 3 + 5 \cdot 2x - 5 \cdot x^2 = 5(3 + 2x - x^2)$$

In some polynomials, pairs of terms have a common binomial factor that can be removed in a process called **factoring by grouping**.

EXAMPLE 2 Factor: $x^3 + 3x^2 - 5x - 15$.

$$\begin{aligned} x^3 + 3x^2 - 5x - 15 &= (x^3 + 3x^2) + (-5x - 15) \\ &= x^2(x + 3) - 5(x + 3) \\ &= (x + 3)(x^2 - 5) \end{aligned}$$

Some trinomials can be factored into the product of two binomials. To factor a trinomial of the form $x^2 + bx + c$, we look for binomial factors of the form $(x + p)(x + q)$, where $p \cdot q = c$ and $p + q = b$. That is, we look for two numbers p and q whose sum is the coefficient of the middle term of the polynomial, b, and whose product is the constant term, c.

EXAMPLES Factor.

3. $x^2 + 5x + 6 = (x + 2)(x + 3)$
4. $x^4 - 6x^3 + 8x^2 = x^2(x^2 - 6x + 8) = x^2(x - 2)(x - 4)$

> To factor trinomials of the type $ax^2 + bx + c, a \neq 1$, using the **FOIL method:**
>
> 1. Factor out the largest common factor.
> 2. Find two First terms whose product is ax^2:
>
> $$(\square x + \quad)(\square x + \quad) = ax^2 + bx + c.$$
>
> **F**OIL
>
> 3. Find two Last terms whose product is c:
>
> $$(\quad x + \square)(\quad x + \square) = ax^2 + bx + c$$
>
> FOI**L**
>
> 4. Repeat steps (2) and (3) until a combination is found for which the sum of the Outer product and the Inner product is bx:
>
> $$(\square x + \square)(\square x + \square) = ax^2 + bx + c.$$
>
> I, O — F**OI**L

(*continued*)

EXAMPLES Factor.

5. $3x^2 - 10x - 8 = (3x + 2)(x - 4)$
6. $12y^2 + 44y - 45 = (2y + 9)(6y - 5)$
7. $r^2 - 7rs + 6s^2 = (r - 6s)(r - s)$
8. $y^4 - 3y^2 - 40 = (y^2 + 5)(y^2 - 8)$

Do Exercises 1–12.

Factor out the largest common factor.

1. $3x + 18$
2. $2z^3 - 8z^2$

Factor by grouping.

3. $3x^3 - x^2 + 18x - 6$
4. $t^3 + 6t^2 - 2t - 12$

Factor the trinomial.

5. $w^2 - 7w + 10$
6. $t^2 + 8t + 15$
7. $2n^2 - 20n - 48$
8. $y^4 - 9y^3 + 14y^2$
9. $2n^2 + 9n - 56$
10. $2y^2 + y - 6$
11. $b^2 - 6bt + 5t^2$
12. $x^4 - 7x^2 - 30$

Just in Time 15

FACTOR POLYNOMIALS: THE *ac*-METHOD

A second method of factoring trinomials of the type $ax^2 + bx + c$, $a \neq 1$, is known as the ***ac*-method**, or the **grouping method**.

THE *ac*-METHOD FOR FACTORING TRINOMIALS

1. Factor out the largest common factor. The remaining trinomial is $ax^2 + bx + c$.
2. Multiply the leading coefficient a and the constant c.
3. Try to factor the product ac so that the sum of the factors is b. That is, find integers p and q such that $pq = ac$ and $p + q = b$.
4. Split the middle term, writing it as a sum using the factors found in step (3).
5. Factor by grouping.

EXAMPLE Factor: $6x^2 + 23x + 20$.

There is no common factor other than 1 or -1. We multiply the leading coefficient, 6, and the constant, 20: $6 \cdot 20 = 120$. Then we look for a factorization of 120 in which the sum of the factors is the coefficient of the middle term, 23. That factorization is $8 \cdot 15$.

(continued)

Split the middle term: $23x = 8x + 15x$.

Factor by grouping:

$$\begin{aligned} 6x^2 + 23x + 20 &= 6x^2 + 8x + 15x + 20 \\ &= 2x(3x + 4) + 5(3x + 4) \\ &= (3x + 4)(2x + 5). \end{aligned}$$

Do Exercises 1–3.

Factor.

1. $8x^2 - 6x - 9$

2. $10t^2 + 4t - 6$

3. $18a^2 - 51a + 15$

Just in Time 16

SPECIAL FACTORIZATIONS

SPECIAL FACTORIZATIONS

- Trinomial Squares:

  $A^2 + 2AB + B^2 = (A + B)^2$;

  $A^2 - 2AB + B^2 = (A - B)^2$

- Difference of Squares:

  $A^2 - B^2 = (A + B)(A - B)$

- Sum or Difference of Cubes:

  $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$;

  $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

EXAMPLES

1. $x^2 - 16 = x^2 - 4^2 = (x + 4)(x - 4)$

2. $$\begin{aligned} x^2 + 8x + 16 &= x^2 + 2 \cdot x \cdot 4 + 4^2 \\ &= (x + 4)^2 \end{aligned}$$

3. $$\begin{aligned} 25y^2 - 30y + 9 &= (5y)^2 - 2 \cdot 5y \cdot 3 + 3^2 \\ &= (5y - 3)^2 \end{aligned}$$

4. $x^3 + 27 = x^3 + 3^3 = (x + 3)(x^2 - 3x + 9)$

5. $$\begin{aligned} 16y^3 - 250 &= 2(8y^3 - 125) \\ &= 2[(2y)^3 - 5^3] \\ &= 2(2y - 5)(4y^2 + 10y + 25) \end{aligned}$$

Do Exercises 1–10.

Factor the difference of squares.

1. $z^2 - 81$

2. $16x^2 - 9$

3. $7pq^4 - 7py^4$

Factor the square of a binomial.

4. $x^2 + 12x + 36$

5. $9z^2 - 12z + 4$

6. $a^3 + 24a^2 + 144a$

Factor the sum or the difference of cubes.

7. $x^3 + 64$

8. $m^3 - 216$

9. $3a^5 - 24a^2$

10. $t^6 + 1$

Just in Time 17

EQUATION-SOLVING PRINCIPLES

For any real numbers a, b, and c,

THE ADDITION PRINCIPLE

If $a = b$ is true, then $a + c = b + c$ is true.

THE MULTIPLICATION PRINCIPLE

If $a = b$ is true, then $ac = bc$ is true.

EXAMPLES Solve.

1.
$$y - 11 = 12$$
$$y - 11 + 11 = 12 + 11$$
$$y = 23$$

Check:
$$\begin{array}{c|l} y - 11 & = 12 \\ \hline 23 - 11 & ?\ 12 \\ 12 & 12 \quad \text{TRUE} \end{array}$$

The solution is 23.

2.
$$15c = 90$$
$$\frac{1}{15} \cdot 15c = \frac{1}{15} \cdot 90$$
$$c = 6$$

The solution is 6.

Check:
$$\begin{array}{c|l} 15c & = 90 \\ \hline 15 \cdot 6 & ?\ 90 \\ 90 & 90 \quad \text{TRUE} \end{array}$$

3.
$$\frac{1}{4}x + 5 = 8$$
$$\frac{1}{4}x + 5 - 5 = 8 - 5$$
$$\frac{1}{4}x = 3$$
$$4 \cdot \frac{1}{4}x = 4 \cdot 3$$
$$x = 12$$

Check:
$$\begin{array}{c|l} \frac{1}{4}x + 5 & = 8 \\ \hline \frac{1}{4} \cdot 12 + 5 & ?\ 8 \\ 3 + 5 & \\ 8 & 8 \quad \text{TRUE} \end{array}$$

The solution is 12.

4.
$$2x + 3 = 1 - 6(x - 1)$$
$$2x + 3 = 1 - 6x + 6$$
$$2x + 3 = 7 - 6x$$
$$2x + 3 + 6x = 7 - 6x + 6x$$
$$8x + 3 = 7$$
$$8x + 3 - 3 = 7 - 3$$
$$8x = 4$$
$$\frac{8x}{8} = \frac{4}{8}$$
$$x = \frac{1}{2}$$

Check:
$$\begin{array}{c|l} 2x + 3 & = 1 - 6(x - 1) \\ \hline 2\left(\frac{1}{2}\right) + 3 & ?\ 1 - 6\left(\frac{1}{2} - 1\right) \\ 1 + 3 & 1 + 3 \\ 4 & 4 \quad \text{TRUE} \end{array}$$

The solution is $\frac{1}{2}$.

Do Exercises 1–8.

Solve.

1. $7t = 70$

2. $x - 5 = 7$

3. $3x + 4 = -8$

4. $6x - 15 = 45$

5. $7y - 1 = 23 - 5y$

6. $3m - 7 = -13 + m$

7. $2(x + 7) = 5x + 14$

8. $5y - 4(2y - 10) = 25$

Just in Time 18

INEQUALITY-SOLVING PRINCIPLES

For any real numbers a, b, and c:

THE ADDITION PRINCIPLE FOR INEQUALITIES

If $a < b$ is true, then $a + c < b + c$ is true.

THE MULTIPLICATION PRINCIPLE FOR INEQUALITIES

a) If $a < b$ and $c > 0$ are true, then $ac < bc$ is true.

b) If $a < b$ and $c < 0$ are true, then $ac > bc$ is true.

(When both sides of an inequality are multiplied by a negative number, the inequality symbol must be reversed.)

Similar statements hold for $a \leq b$.

EXAMPLES Solve.

1.
$$a + 9 \leq -50$$
$$a + 9 - 9 \leq -50 - 9$$
$$a \leq -59$$

The solution set is $(-\infty, -59]$.

2.
$$-5x < 4$$
$$-\tfrac{1}{5}(-5x) > -\tfrac{1}{5}(4)$$
$$x > -\tfrac{4}{5}$$

The solution set is $\left(-\tfrac{4}{5}, \infty\right)$.

3.
$$2y - 1 < 5$$
$$2y - 1 + 1 < 5 + 1$$
$$2y < 6$$
$$\tfrac{1}{2} \cdot 2y < \tfrac{1}{2} \cdot 6$$
$$y < 3$$

The solution set is $(-\infty, 3)$.

4.
$$4 - 3x \geq 13$$
$$-4 + 4 - 3x \geq -4 + 13$$
$$-3x \geq 9$$
$$\frac{-3x}{-3} \leq \frac{9}{-3}$$
$$x \leq -3$$

The solution set is $(-\infty, -3]$.

Do Exercises 1–6.

Solve.

1. $p + 25 \geq -100$

2. $-\frac{2}{3}x > 6$

3. $9x - 1 < 17$

4. $-x - 16 \geq 40$

5. $\frac{1}{3}y - 6 < 3$

6. $8 - 2w \leq -14$

Just in Time 19

THE PRINCIPLE OF ZERO PRODUCTS

The product of two numbers is 0 if one or both of the numbers is 0. Furthermore, *if any product is* 0, *then a factor must be* 0. For example:

If $7x = 0$, then we know that $x = 0$.

If $x(2x - 9) = 0$, then we know that $x = 0$ *or* $2x - 9 = 0$.

If $(x + 3)(x - 2) = 0$, then we know that $x + 3 = 0$ *or* $x - 2 = 0$.

> **THE PRINCIPLE OF ZERO PRODUCTS**
>
> If $ab = 0$ is true, then $a = 0$ or $b = 0$, and if $a = 0$ or $b = 0$, then $ab = 0$.

Some quadratic equations can be solved using the principle of zero products.

EXAMPLES Solve.

1. $(2q - 7)(q + 4) = 0$

$2q - 7 = 0$ *or* $q + 4 = 0$

$2q = 7$ *or* $q = -4$

$q = \frac{7}{2}$ *or* $q = -4$

The solutions are -4 and $\frac{7}{2}$.

2. $5x^2 - 75x = 0$

$5x(x - 15) = 0$

$5x = 0$ *or* $x - 15 = 0$

$x = 0$ *or* $x = 15$

The solutions are 0 and 15.

3. $x^2 - 3x - 4 = 0$

$(x + 1)(x - 4) = 0$

$x + 1 = 0$ *or* $x - 4 = 0$

$x = -1$ *or* $x = 4$

The solutions are -1 and 4.

4. $2x^2 + 5x - 3 = 0$

$(x + 3)(2x - 1) = 0$

$x + 3 = 0$ *or* $2x - 1 = 0$

$x = -3$ *or* $2x = 1$

$x = -3$ *or* $x = \frac{1}{2}$

The solutions are -3 and $\frac{1}{2}$.

Do Exercises 1–8.

Solve.

1. $2y^2 + 42y = 0$

2. $(a + 7)(a - 1) = 0$

3. $(5y + 3)(y - 4) = 0$

4. $6x^2 + 7x - 5 = 0$

5. $t(t - 8) = 0$

6. $x^2 - 8x - 33 = 0$

7. $x^2 + 13x = 30$

8. $12x^2 - 7x - 12 = 0$

Just in Time 20

THE PRINCIPLE OF SQUARE ROOTS

The principle of square roots can be used to solve some quadratic equations.

> **THE PRINCIPLE OF SQUARE ROOTS**
> If $x^2 = k$, then $x = -\sqrt{k}$ *or* $x = \sqrt{k}$.

EXAMPLES Solve.

1.
$$s^2 - 144 = 0$$
$$s^2 = 144$$
$$s = -\sqrt{144} \quad or \quad s = \sqrt{144}$$
$$s = -12 \quad or \quad s = 12$$

The solutions are -12 and 12, or ± 12.

2.
$$3x^2 - 21 = 0$$
$$3x^2 = 21$$
$$x^2 = 7$$
$$x = -\sqrt{7} \quad or \quad x = \sqrt{7}$$

The solutions are $-\sqrt{7}$ and $\sqrt{7}$, or $\pm\sqrt{7}$.

Do Exercises 1–6.

Solve.

1. $x^2 - 36 = 0$

2. $2y^2 - 20 = 0$

3. $6z^2 = 18$

4. $3t^2 - 15 = 0$

5. $z^2 - 1 = 24$

6. $5x^2 - 75 = 0$

Just in Time 21

SIMPLIFY RATIONAL EXPRESSIONS

A **rational expression** is the quotient of two polynomials. The **domain** of an algebraic expression is the set of all real numbers for which the expression is defined. Since division by 0 is not defined, any number that makes the denominator 0 is not in the domain of a rational expression.

EXAMPLE 1 Find the domain of

$$\frac{x^2 - 4}{x^2 - 4x - 5}.$$

We solve the equation $x^2 - 4x - 5 = 0$, or $(x + 1)(x - 5) = 0$, to find the numbers that are not in the domain. The solutions are -1 and 5. Since the denominator is 0 when $x = -1$ *or* $x = 5$, the domain is the set of all real numbers except -1 and 5.

EXAMPLE 2 Simplify: $\dfrac{9x^2 + 6x - 3}{12x^2 - 12}$.

$$\begin{aligned}\frac{9x^2 + 6x - 3}{12x^2 - 12} &= \frac{3(3x^2 + 2x - 1)}{12(x^2 - 1)}\\ &= \frac{3(x + 1)(3x - 1)}{3 \cdot 4(x + 1)(x - 1)}\\ &= \frac{3(x + 1)}{3(x + 1)} \cdot \frac{3x - 1}{4(x - 1)}\\ &= 1 \cdot \frac{3x - 1}{4(x - 1)}\\ &= \frac{3x - 1}{4(x - 1)}\end{aligned}$$

Canceling is a shortcut that is often used to remove a factor of 1.

EXAMPLE 3 Simplify: $\dfrac{2 - x}{x^2 + x - 6}$.

$$\begin{aligned}\frac{2 - x}{x^2 + x - 6} &= \frac{2 - x}{(x + 3)(x - 2)}\\ &= \frac{-1(x - 2)}{(x + 3)(x - 2)}\\ &= \frac{-1\cancel{(x - 2)}}{(x + 3)\cancel{(x - 2)}}\\ &= \frac{-1}{x + 3}, \text{ or } -\frac{1}{x + 3}\end{aligned}$$

Do Exercises 1–6.

Find the domain of the rational expression.

1. $\dfrac{3x - 3}{x(x - 1)}$

2. $\dfrac{y + 6}{y^2 + 4y - 21}$

Simplify.

3. $\dfrac{x^2 - 4}{x^2 - 4x + 4}$

4. $\dfrac{x^2 + 2x - 3}{x^2 - 9}$

5. $\dfrac{x^3 - 6x^2 + 9x}{x^3 - 3x^2}$

6. $\dfrac{6y^2 + 12y - 48}{3y^2 - 9y + 6}$

Just in Time 22

MULTIPLY AND DIVIDE RATIONAL EXPRESSIONS

To multiply rational expressions, we multiply numerators and multiply denominators and, if possible, simplify the result. To divide rational expressions, we multiply the dividend by the reciprocal of the divisor and, if possible, simplify the result; that is,

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd} \quad \text{and} \quad \frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}.$$

EXAMPLES Multiply or divide.

1. $$\frac{a^2-4}{16a}\cdot\frac{20a^2}{a+2}=\frac{(a^2-4)(20a^2)}{16a(a+2)}$$
$$=\frac{\cancel{(a+2)}(a-2)\cdot\cancel{4}\cdot 5\cdot a\cdot\cancel{a}}{\cancel{4}\cdot 4\cdot\cancel{a}\cdot\cancel{(a+2)}}$$
$$=\frac{5a(a-2)}{4}$$

2. $$\frac{x-2}{12}\div\frac{x^2-4x+4}{3x^3+15x^2}=\frac{x-2}{12}\cdot\frac{3x^3+15x^2}{x^2-4x+4}$$
$$=\frac{(x-2)(3x^3+15x^2)}{12(x^2-4x+4)}$$
$$=\frac{\cancel{(x-2)}(\cancel{3})(x^2)(x+5)}{\cancel{3}\cdot 4\,\cancel{(x-2)}(x-2)}$$
$$=\frac{x^2(x+5)}{4(x-2)}$$

Do Exercises 1–6.

Multiply or divide and, if possible, simplify.

1. $\dfrac{r-s}{r+s}\cdot\dfrac{r^2-s^2}{(r-s)^2}$

2. $\dfrac{m^2-n^2}{r+s}\div\dfrac{m-n}{r+s}$

3. $\dfrac{4x^2+9x+2}{x^2+x-2}\cdot\dfrac{x^2-1}{3x^2+x-2}$

4. $\dfrac{a^2-a-2}{a^2-a-6}\div\dfrac{a^2-2a}{2a+a^2}$

5. $\dfrac{3x+12}{2x-8}\div\dfrac{(x+4)^2}{(x-4)^2}$

6. $\dfrac{x^2-y^2}{x^3-y^3}\cdot\dfrac{x^2+xy+y^2}{x^2+2xy+y^2}$

Just in Time 23

ADD AND SUBTRACT RATIONAL EXPRESSIONS

When rational expressions have the same denominator, we can add or subtract by adding or subtracting the numerators and retaining the common denominator. If the denominators differ, we must find equivalent rational expressions that have a common denominator before we can add or subtract. In general, it is most efficient to find the **least common denominator (LCD)** of the expressions.

To find the least common denominator of rational expressions, factor each denominator and form the product that uses each factor the greatest number of times it occurs in any factorization.

EXAMPLE 1 Add.

$$\frac{x^2 - 4x + 4}{2x^2 - 3x + 1} + \frac{x + 4}{2x - 2}$$

$$= \frac{x^2 - 4x + 4}{(2x - 1)(x - 1)} + \frac{x + 4}{2(x - 1)}$$

The LCD is $(2x - 1)(x - 1)(2)$, or $2(2x - 1)(x - 1)$.

$$= \frac{x^2 - 4x + 4}{(2x - 1)(x - 1)} \cdot \frac{2}{2} + \frac{x + 4}{2(x - 1)} \cdot \frac{2x - 1}{2x - 1}$$

$$= \frac{2x^2 - 8x + 8}{(2x - 1)(x - 1)(2)} + \frac{2x^2 + 7x - 4}{2(x - 1)(2x - 1)}$$

$$= \frac{4x^2 - x + 4}{2(2x - 1)(x - 1)}$$

EXAMPLE 2 Subtract.

$$\frac{x}{x^2 + 11x + 30} - \frac{5}{x^2 + 9x + 20}$$

$$= \frac{x}{(x + 5)(x + 6)} - \frac{5}{(x + 5)(x + 4)}$$

The LCD is $(x + 5)(x + 6)(x + 4)$.

$$= \frac{x}{(x + 5)(x + 6)} \cdot \frac{x + 4}{x + 4} - \frac{5}{(x + 5)(x + 4)} \cdot \frac{x + 6}{x + 6}$$

$$= \frac{x^2 + 4x}{(x + 5)(x + 6)(x + 4)} - \frac{5x + 30}{(x + 5)(x + 4)(x + 6)}$$

$$= \frac{x^2 + 4x - (5x + 30)}{(x + 5)(x + 6)(x + 4)} = \frac{x^2 + 4x - 5x - 30}{(x + 5)(x + 6)(x + 4)}$$

$$= \frac{x^2 - x - 30}{(x + 5)(x + 6)(x + 4)} = \frac{\cancel{(x + 5)}(x - 6)}{\cancel{(x + 5)}(x + 6)(x + 4)}$$

$$= \frac{x - 6}{(x + 6)(x + 4)}$$

Do Exercises 1–6.

Add or subtract and, if possible, simplify.

1. $\frac{a - 3b}{a + b} + \frac{a + 5b}{a + b}$

2. $\frac{x^2 - 5}{3x^2 - 5x - 2} + \frac{x + 1}{3x - 6}$

3. $\frac{a^2 + 1}{a^2 - 1} - \frac{a - 1}{a + 1}$

4. $\frac{9x + 2}{3x^2 - 2x - 8} + \frac{7}{3x^2 + x - 4}$

5. $\frac{y}{y^2 - y - 20} - \frac{2}{y + 4}$

6. $\frac{3y}{y^2 - 7y + 10} - \frac{2y}{y^2 - 8y + 15}$

Just in Time 24

SIMPLIFY COMPLEX RATIONAL EXPRESSIONS

A **complex rational expression** has rational expressions in its numerator or its denominator or both.

EXAMPLE Simplify: $\dfrac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a^3}+\dfrac{1}{b^3}}$.

Method 1:

$$\frac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a^3}+\dfrac{1}{b^3}} = \frac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a^3}+\dfrac{1}{b^3}} \cdot \frac{a^3b^3}{a^3b^3}$$

The LCD of the four rational expressions in the numerator and the denominator is a^3b^3.

$$= \frac{\left(\dfrac{1}{a}+\dfrac{1}{b}\right)(a^3b^3)}{\left(\dfrac{1}{a^3}+\dfrac{1}{b^3}\right)(a^3b^3)} = \frac{a^2b^3 + a^3b^2}{b^3 + a^3}$$

$$= \frac{a^2b^2(\cancel{b+a})}{(\cancel{b+a})(b^2 - ba + a^2)} = \frac{a^2b^2}{b^2 - ba + a^2}$$

Method 2:

$$\frac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a^3}+\dfrac{1}{b^3}} = \frac{\dfrac{1}{a}\cdot\dfrac{b}{b}+\dfrac{1}{b}\cdot\dfrac{a}{a}}{\dfrac{1}{a^3}\cdot\dfrac{b^3}{b^3}+\dfrac{1}{b^3}\cdot\dfrac{a^3}{a^3}}$$

The LCD is ab. The LCD is a^3b^3.

$$= \frac{\dfrac{b}{ab}+\dfrac{a}{ab}}{\dfrac{b^3}{a^3b^3}+\dfrac{a^3}{a^3b^3}}$$

$$= \frac{\dfrac{b+a}{ab}}{\dfrac{b^3+a^3}{a^3b^3}} = \frac{b+a}{ab}\cdot\frac{a^3b^3}{b^3+a^3}$$

$$= \frac{(\cancel{b+a})(\cancel{ab})(a^2b^2)}{(\cancel{ab})(\cancel{b+a})(b^2 - ba + a^2)}$$

$$= \frac{a^2b^2}{b^2 - ba + a^2}$$

Do Exercises 1–5.

Simplify.

1. $\dfrac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{1}{y}+\dfrac{1}{x}}$

2. $\dfrac{\dfrac{a-b}{b}}{\dfrac{a^2-b^2}{ab}}$

3. $\dfrac{w+\dfrac{8}{w^2}}{1+\dfrac{2}{w}}$

4. $\dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{x-y}{y}}$

5. $\dfrac{\dfrac{a}{b}-\dfrac{b}{a}}{\dfrac{1}{a}-\dfrac{1}{b}}$

Note: $b - a = -1(a - b)$.

Just in Time 25

SIMPLIFY RADICAL EXPRESSIONS

The symbol $\sqrt{a}$ denotes the nonnegative square root of a, and the symbol $\sqrt[3]{a}$ denotes the real-number cube root of a. The symbol $\sqrt[n]{a}$ denotes the nth root of a; that is, a number whose nth power is a. The symbol $\sqrt[n]{\ }$ is called a **radical**, and the expression under the radical is called the **radicand**. The number n (which is omitted when it is 2) is called the **index**.

Any positive number has two square roots, one positive and one negative. Similarly, for any even index, a positive number has two real-number roots. The positive root is called the **principal root**. Any real number has only one real-number odd root.

EXAMPLES Simplify.

1. $\sqrt{36} = 6$ because $6 \cdot 6 = 36$.
2. $-\sqrt{36} = -6$
3. $\sqrt[3]{-8} = -2$
4. $\sqrt[5]{\dfrac{32}{243}} = \dfrac{2}{3}$
5. $\sqrt[4]{-16}$ is not a real number.

PROPERTIES OF RADICALS

Let a and b be any real numbers or expressions for which the given roots exist. For any natural numbers m and n $(n \neq 1)$:

1. If n is even, $\sqrt[n]{a^n} = |a|$.
2. If n is odd, $\sqrt[n]{a^n} = a$.
3. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.
4. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $(b \neq 0)$.
5. $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$.

Here, we assume that no radicands are formed by raising negative quantities to even powers and, consequently, we will not use absolute-value notation when we simplify radical expressions involving variables.

EXAMPLES Simplify.

6. $\sqrt{(-5)^2} = |-5| = 5$
7. $\sqrt[3]{(-5)^3} = -5$
8. $\sqrt[4]{4} \cdot \sqrt[4]{5} = \sqrt[4]{4 \cdot 5} = \sqrt[4]{20}$
9. $\sqrt[3]{8^5} = (\sqrt[3]{8})^5 = 2^5 = 32$
10. $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$
11. $\dfrac{\sqrt{72}}{\sqrt{6}} = \sqrt{\dfrac{72}{6}} = \sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$
12. $\sqrt{216x^5y^3} = \sqrt{36 \cdot 6 \cdot x^4 \cdot x \cdot y^2 \cdot y} = \sqrt{36x^4y^2}\sqrt{6xy}$
 $= 6x^2y\sqrt{6xy}$
13. $8\sqrt{50} - 3\sqrt{8} = 8\sqrt{25 \cdot 2} - 3\sqrt{4 \cdot 2} = 8 \cdot 5\sqrt{2} - 3 \cdot 2\sqrt{2}$
 $= 40\sqrt{2} - 6\sqrt{2} = (40 - 6)\sqrt{2} = 34\sqrt{2}$
14. $(5 - \sqrt{2})(4 + 3\sqrt{2}) = 20 + 15\sqrt{2} - 4\sqrt{2} - 3(\sqrt{2})^2$
 $= 20 + 11\sqrt{2} - 6 = 14 + 11\sqrt{2}$

Do Exercises 1–20.

Simplify. Assume that no radicands were formed by raising negative quantities to even powers.

1. $\sqrt{(-21)^2}$
2. $\sqrt{9y^2}$
3. $\sqrt{(a-2)^2}$
4. $\sqrt[3]{-27x^3}$
5. $\sqrt[4]{81x^8}$
6. $\sqrt[5]{32}$
7. $\sqrt[4]{48x^6y^4}$
8. $\sqrt{15}\sqrt{35}$
9. $\dfrac{\sqrt{40xy}}{\sqrt{8x}}$
10. $\dfrac{\sqrt[3]{3x^2}}{\sqrt[3]{24x^5}}$
11. $\sqrt{x^2 - 4x + 4}$
12. $\sqrt{2x^3y}\sqrt{12xy}$
13. $\sqrt[3]{3x^2y}\sqrt[3]{36x}$
14. $5\sqrt{2} + 3\sqrt{32}$
15. $7\sqrt{12} - 2\sqrt{3}$
16. $2\sqrt{32} + 3\sqrt{8} - 4\sqrt{18}$
17. $6\sqrt{20} - 4\sqrt{45} + \sqrt{80}$
18. $(2 + \sqrt{3})(5 + 2\sqrt{3})$
19. $(\sqrt{8} + 2\sqrt{5})(\sqrt{8} - 2\sqrt{5})$
20. $(1 + \sqrt{3})^2$

Just in Time 26

RATIONALIZE DENOMINATORS

There are times when we need to remove the radicals in a denominator. This procedure is called **rationalizing the denominator**. It is done by multiplying by 1 in such a way as to obtain a perfect *n*th power in the denominator.

EXAMPLES Rationalize the denominator.

1. $\sqrt{\frac{3}{2}} = \sqrt{\frac{3}{2} \cdot \frac{2}{2}} = \sqrt{\frac{6}{4}} = \frac{\sqrt{6}}{\sqrt{4}} = \frac{\sqrt{6}}{2}$

2. $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

3. $\frac{\sqrt[3]{7}}{\sqrt[3]{9}} = \frac{\sqrt[3]{7}}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{\sqrt[3]{21}}{\sqrt[3]{27}} = \frac{\sqrt[3]{21}}{3}$

Pairs of expressions of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ are called **conjugates**. The product of such a pair contains no radicals and can be used to rationalize a denominator or a numerator.

EXAMPLE 4 Rationalize the denominator: $\frac{7}{3 + \sqrt{5}}$.

$$\frac{7}{3 + \sqrt{5}} = \frac{7}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}}$$
$$= \frac{21 - 7\sqrt{5}}{3^2 - 3\sqrt{5} + 3\sqrt{5} - (\sqrt{5})^2}$$
$$= \frac{21 - 7\sqrt{5}}{9 - 5} = \frac{21 - 7\sqrt{5}}{4}$$

EXAMPLE 5 Rationalize the denominator: $\frac{1 + \sqrt{2}}{\sqrt{5} + \sqrt{10}}$.

$$\frac{1 + \sqrt{2}}{\sqrt{5} + \sqrt{10}} = \frac{1 + \sqrt{2}}{\sqrt{5} + \sqrt{10}} \cdot \frac{\sqrt{5} - \sqrt{10}}{\sqrt{5} - \sqrt{10}}$$
$$= \frac{\sqrt{5} - \sqrt{10} + \sqrt{10} - \sqrt{20}}{(\sqrt{5})^2 - (\sqrt{10})^2}$$
$$= \frac{\sqrt{5} - \sqrt{20}}{5 - 10}$$
$$= \frac{\sqrt{5} - 2\sqrt{5}}{-5}$$
$$= \frac{-\sqrt{5}}{-5}$$
$$= \frac{\sqrt{5}}{5}$$

Do Exercises 1–8.

Rationalize the denominator.

1. $\frac{4}{\sqrt{11}}$

2. $\sqrt{\frac{3}{7}}$

3. $\frac{\sqrt[3]{7}}{\sqrt[3]{2}}$

4. $\sqrt[3]{\frac{16}{9}}$

5. $\frac{3}{\sqrt{30} - 4}$

6. $\frac{4}{\sqrt{7} - \sqrt{3}}$

7. $\frac{6}{\sqrt{m} - \sqrt{n}}$

8. $\frac{1 - \sqrt{2}}{\sqrt{3} - \sqrt{6}}$

Just in Time 27

RATIONAL EXPONENTS

For any real number a and any natural numbers m and n, $n \neq 1$, for which $\sqrt[n]{a}$ exists:

$$a^{1/n} = \sqrt[n]{a}, \qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}, \quad \text{and} \quad a^{-m/n} = \frac{1}{a^{m/n}}.$$

EXAMPLES Convert to radical notation and, if possible, simplify.

1. $m^{1/6} = \sqrt[6]{m}$

2. $7^{3/4} = \sqrt[4]{7^3}$, or $\left(\sqrt[4]{7}\right)^3$

3. $8^{-5/3} = \dfrac{1}{8^{5/3}} = \dfrac{1}{\left(\sqrt[3]{8}\right)^5} = \dfrac{1}{2^5} = \dfrac{1}{32}$

EXAMPLES Convert to exponential notation.

4. $\left(\sqrt[4]{7xy}\right)^5 = (7xy)^{5/4}$

5. $\sqrt[6]{x^3} = x^{3/6} = x^{1/2}$

EXAMPLES Simplify and then, if appropriate, write radical notation.

6. $x^{5/6} \cdot x^{2/3} = x^{5/6 + 2/3} = x^{9/6} = x^{3/2} = \sqrt{x^3}$
$= \sqrt{x^2}\sqrt{x} = x\sqrt{x}$

7. $(x + 3)^{5/2}(x + 3)^{-1/2} = (x + 3)^{5/2 - 1/2} = (x + 3)^2$

Do Exercises 1–11.

Convert to radical notation and, if possible, simplify.

1. $y^{5/6}$

2. $x^{2/3}$

3. $16^{3/4}$

4. $4^{7/2}$

5. $125^{-1/3}$

6. $32^{-4/5}$

Convert to exponential notation.

7. $\sqrt[12]{y^4}$

8. $\sqrt{x^5}$

Simplify and then, if appropriate, write radical notation.

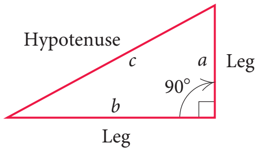
9. $x^{1/2} \cdot x^{2/3}$

10. $(a - 2)^{9/4}(a - 2)^{-1/4}$

11. $(m^{1/2}n^{5/2})^{2/3}$

Just in Time 28 THE PYTHAGOREAN THEOREM

A **right triangle** is a triangle with a 90° angle, as shown in the following figure. The small square in the corner indicates the 90° angle.

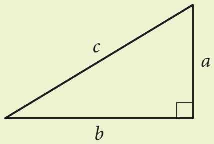


In a right triangle, the longest side is called the **hypotenuse**. It is also the side opposite the right angle. The other two sides are called **legs**. We generally use the letters a and b for the lengths of the legs and c for the length of the hypotenuse. They are related as follows.

THE PYTHAGOREAN THEOREM

In any right triangle, if a and b are the lengths of the legs and c is the length of the hypotenuse, then

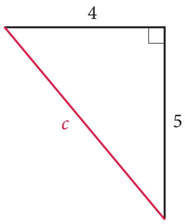
$$a^2 + b^2 = c^2.$$



The equation $a^2 + b^2 = c^2$ is called the **Pythagorean equation**.

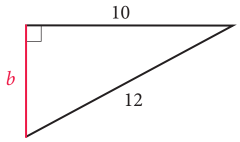
EXAMPLE 1 Find the length of the hypotenuse of this right triangle. Give an exact answer and an approximation to three decimal places.

$$4^2 + 5^2 = c^2$$
$$16 + 25 = c^2$$
$$41 = c^2$$
$$c = \sqrt{41}$$
$$c \approx 6.403$$



EXAMPLE 2 Find the length of leg b of this right triangle. Give an exact answer and an approximation to three decimal places.

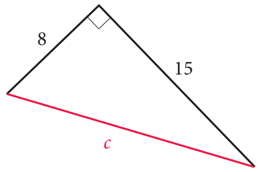
$$10^2 + b^2 = 12^2$$
$$100 + b^2 = 144$$
$$b^2 = 144 - 100$$
$$b^2 = 44$$
$$b = \sqrt{44}$$
$$b \approx 6.633$$



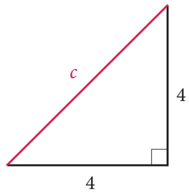
Do Exercises 1–5.

Find the length of the third side of each right triangle. Where appropriate, give both an exact answer and an approximation to three decimal places.

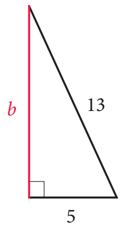
1.



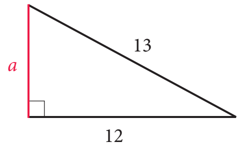
2.



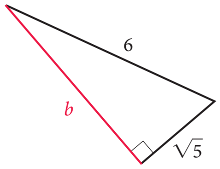
3.



4.



5.



CHAPTER

1

Graphs, Functions, and Models

APPLICATION

This problem appears as Exercise 60 in the Review Exercises.

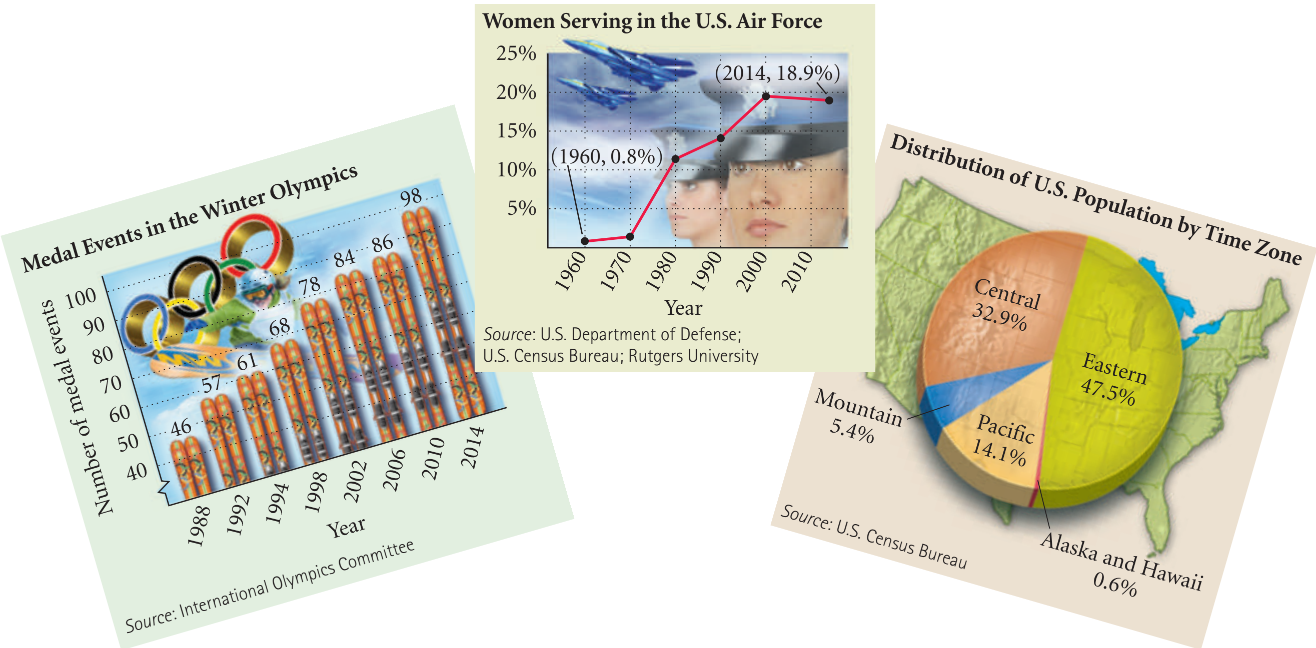
The number of female medical school graduates has increased each year from 2006 to 2014. Model the data shown on p. 88 with a linear function where the number of female medical school graduates W is a function of the year x and where x is the number of years after 2006. Then, using this function, estimate the number of female graduates in 2013.

1.1 Introduction to Graphing

- Plot points.
- Determine whether an ordered pair is a solution of an equation.
- Find the x- and y-intercepts of an equation of the form $Ax + By = C$.
- Graph equations.
- Find the distance between two points in the plane, and find the midpoint of a segment.
- Find an equation of a circle with a given center and radius, and given an equation of a circle in standard form, find the center and the radius.
- Graph equations of circles.

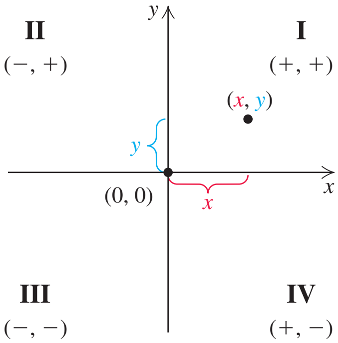
Graphs

Graphs provide a means of displaying, interpreting, and analyzing data in a visual format. It is not uncommon to open a newspaper or a magazine and encounter graphs. Examples of bar, line, and circle graphs are shown below.



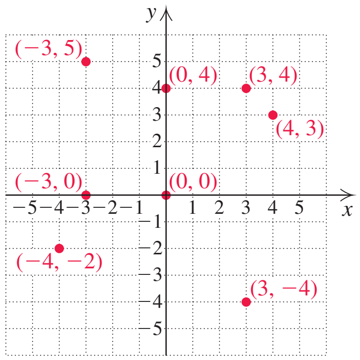
Many real-world situations can be modeled, or described mathematically, using equations in which two variables appear. We use a plane to graph a pair of numbers. To locate points on a plane, we use two perpendicular number lines, called **axes**, that intersect at $(0, 0)$. We call this point the **origin**. The horizontal axis is called the ***x*-axis**, and the vertical axis is called the ***y*-axis**. (Other variables, such as a and b, can also be used.) The axes divide the plane into four regions, called **quadrants**, denoted by Roman numerals and numbered counterclockwise from the upper right. Arrows show the positive direction of each axis.

Each point (x, y) in the plane is described by an **ordered pair**. The first number, x, indicates the point's horizontal location with respect to the y-axis, and the



second number, y, indicates the point's vertical location with respect to the x-axis. We call x the **first coordinate,** the ***x*-coordinate**, or the **abscissa**. We call y the **second coordinate,** the ***y*-coordinate**, or the **ordinate**. Such a representation is called the **Cartesian coordinate system** in honor of the French mathematician and philosopher René Descartes (1596–1650).

In the first quadrant, both coordinates of a point are positive. In the second quadrant, the first coordinate is negative and the second is positive. In the third quadrant, both coordinates are negative, and in the fourth quadrant, the first coordinate is positive and the second is negative.



EXAMPLE 1 Graph and label the points $(-3, 5)$, $(4, 3)$, $(3, 4)$, $(-4, -2)$, $(3, -4)$, $(0, 4)$, $(-3, 0)$, and $(0, 0)$.

Solution To graph or **plot** $(-3, 5)$, we note that the x-coordinate, -3, tells us to move from the origin 3 units horizontally in the negative direction, or 3 units to the left of the y-axis. Then we move 5 units up from the x-axis.* To graph the other points, we proceed in a similar manner. (See the graph at left.) Note that the point $(4, 3)$ is different from the point $(3, 4)$.

Now Try Exercise 3.

Solutions of Equations

Equations in two variables, like $2x + 3y = 18$, have solutions (x, y) that are ordered pairs such that when the first coordinate is substituted for x and the second coordinate is substituted for y, the result is a true equation. The first coordinate in an ordered pair generally represents the variable that occurs first alphabetically.

EXAMPLE 2 Determine whether each ordered pair is a solution of the equation $2x + 3y = 18$.

a) $(-5, 7)$ **b)** $(3, 4)$

Solution We substitute the ordered pair into the equation and determine whether the resulting equation is true.

a)

$$\begin{array}{r|l} 2x + 3y & = 18 \\ \hline 2(-5) + 3(7) & ?\ 18 \\ -10 + 21 & \\ 11 & 18 \end{array}$$

We substitute -5 for x and 7 for y (alphabetical order).

FALSE

The equation $11 = 18$ is false, so $(-5, 7)$ is not a solution.

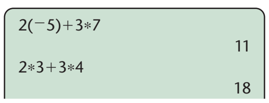
b)

$$\begin{array}{r|l} 2x + 3y & = 18 \\ \hline 2(3) + 3(4) & ?\ 18 \\ 6 + 12 & \\ 18 & 18 \end{array}$$

We substitute 3 for x and 4 for y.

TRUE

The equation $18 = 18$ is true, so $(3, 4)$ is a solution.



We can also perform these substitutions on a graphing calculator. When we substitute -5 for x and 7 for y, we get 11. Since $11 \neq 18$, $(-5, 7)$ is not a solution of the equation. When we substitute 3 for x and 4 for y, we get 18, so $(3, 4)$ is a solution.

Now Try Exercise 11.

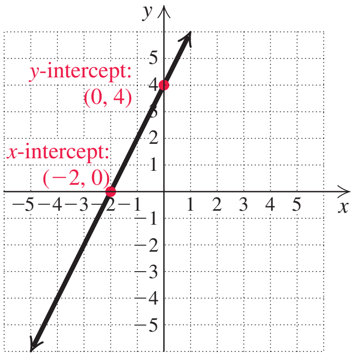
*Here the notation $(-3, 5)$ represents an ordered pair. This notation can also represent an open interval. See Just-in-Time 6 review on p. J-6. The context in which the notation appears usually makes the meaning clear.

Graphs of Equations

The equation considered in Example 2 actually has an infinite number of solutions. Since we cannot list all the solutions, we will make a drawing, called a **graph**, that represents them. Some suggestions for drawing graphs are on the following page.

TO GRAPH AN EQUATION

To **graph an equation** is to make a drawing that represents the solutions of that equation.



Graphs of equations of the type $Ax + By = C$ are straight lines. Many such equations can be graphed conveniently using intercepts. The ***x*-intercept** of the graph of an equation is the point at which the graph crosses the x-axis. The ***y*-intercept** is the point at which the graph crosses the y-axis. We know from geometry that only one line can be drawn through two given points. Thus, if we know the intercepts, we can graph the line. To ensure that a computational error has not been made, it is a good idea to calculate and plot a third point as a check.

***x*-INTERCEPT AND *y*-INTERCEPT**

An ***x*-intercept** is a point $(a, 0)$. To find a, let $y = 0$ and solve for x.

A ***y*-intercept** is a point $(0, b)$. To find b, let $x = 0$ and solve for y.

EXAMPLE 3 Graph: $2x + 3y = 18$.

Solution The graph is a line. To find ordered pairs that are solutions of this equation, we can replace either x or y with any number and then solve for the other variable. In this case, it is convenient to find the intercepts of the graph. For instance, if x is replaced with 0, then

$$2 \cdot 0 + 3y = 18$$
$$3y = 18$$
$$y = 6. \qquad \text{Dividing by 3 on both sides}$$

Thus, $(0, 6)$ is a solution. It is the *y-intercept* of the graph. If y is replaced with 0, then

$$2x + 3 \cdot 0 = 18$$
$$2x = 18$$
$$x = 9. \qquad \text{Dividing by 2 on both sides}$$

Thus, $(9, 0)$ is a solution. It is the *x-intercept* of the graph. We find a third solution as a check. If x is replaced with 3, then

$$2 \cdot 3 + 3y = 18$$
$$6 + 3y = 18$$
$$3y = 12 \qquad \text{Subtracting 6 on both sides}$$
$$y = 4. \qquad \text{Dividing by 3 on both sides}$$

Thus, $(3, 4)$ is a solution.

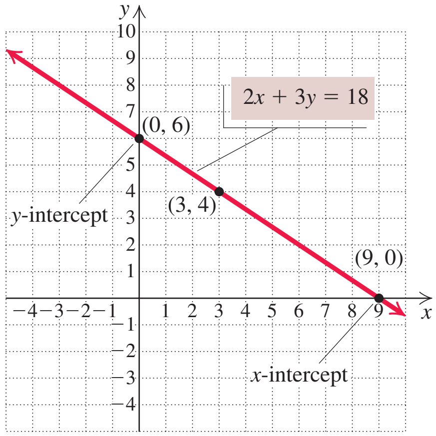
STUDY TIPS

Success can be planned. Combine goals and good study habits to create a plan for success that works for you. A list of study tips that your authors consider most helpful are included in the Guide to Success in the front of the text before Chapter 1.

> **SUGGESTIONS FOR DRAWING GRAPHS**
>
> 1. Calculate solutions and list the ordered pairs in a table.
> 2. Use graph paper.
> 3. Draw axes and label them with the variables.
> 4. Use arrows on the axes to indicate positive directions.
> 5. Scale the axes; that is, label the tick marks on the axes. Consider the ordered pairs found in part (1) above when choosing the scale.
> 6. Plot the ordered pairs, look for patterns, and complete the graph. Label the graph with the equation being graphed.

We list the solutions in a table and then plot the points. Note that the points appear to lie on a straight line.

x	y	(x, y)
0	6	(0, 6)
9	0	(9, 0)
3	4	(3, 4)



Were we to graph additional solutions of $2x + 3y = 18$, they would lie on the same straight line. Thus, to complete the graph, we use a straightedge to draw a line, as shown in the figure. This line represents all solutions of the equation. Every point on the line represents a solution; every solution is represented by a point on the line.

Now Try Exercise 17.

When graphing some equations, it is convenient to first solve for y and then find ordered pairs. We can use the addition and multiplication principles to solve for y.

EXAMPLE 4 Graph: $3x - 5y = -10$.

Solution We first solve for y:

Just in Time 17

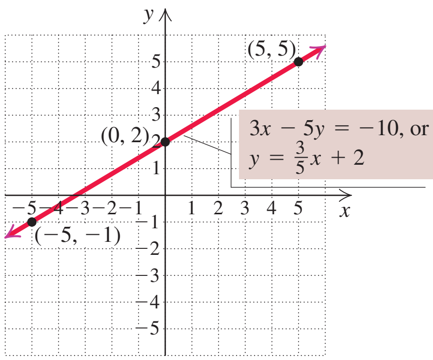
$$3x - 5y = -10$$
$$-5y = -3x - 10 \quad \text{Subtracting } 3x \text{ on both sides}$$
$$y = \tfrac{3}{5}x + 2. \quad \text{Multiplying by } -\tfrac{1}{5} \text{ on both sides}$$

By choosing multiples of 5 for x, we can avoid adding and subtracting fraction values when calculating y. For example, if we choose -5 for x, we get

$$y = \tfrac{3}{5}x + 2 = \tfrac{3}{5}(-5) + 2 = -3 + 2 = -1.$$

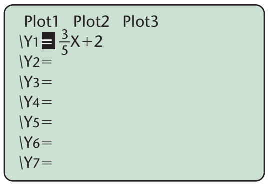
The following table lists a few points. We plot the points and draw the graph.

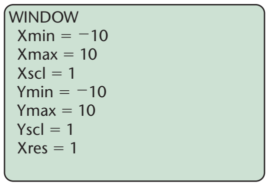
x	y	(x, y)
−5	−1	(−5, −1)
0	2	(0, 2)
5	5	(5, 5)



Now Try Exercise 29.

In the equation $y = \frac{3}{5}x + 2$ in Example 4, the value of y *depends* on the value chosen for x, so x is said to be the **independent variable** and y the **dependent variable**.





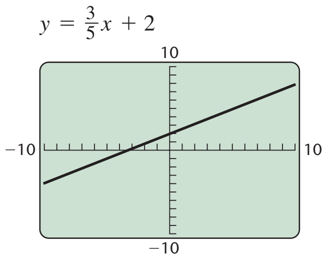
We can graph an equation on a graphing calculator. Many calculators require an equation to be entered in the form $y =$. In such a case, if the equation is not initially given in this form, it must be solved for y before it is entered in the calculator. For the equation $3x - 5y = -10$ in Example 4, we enter $y = \frac{3}{5}x + 2$ on the equation-editor, or $y =$, screen in the form $y = (3/5)x + 2$, which some calculators will display as shown in the window at left.

Next, we determine the portion of the xy-plane that will appear on the calculator's screen. That portion of the plane is called the **viewing window**.

The notation used in this text to denote a window setting consists of four numbers $[L, R, B, T]$, which represent the **L**eft and **R**ight endpoints of the x-axis and the **B**ottom and **T**op endpoints of the y-axis, respectively. The window with the settings $[-10, 10, -10, 10]$ is the **standard viewing window**. On some graphing calculators, the standard window can be selected quickly using the ZSTANDARD feature from the ZOOM menu.

Xmin and Xmax are used to set the left and right endpoints of the x-axis, respectively; Ymin and Ymax are used to set the bottom and top endpoints of the y-axis, respectively. The settings Xscl and Yscl give the scales for the axes. For example, Xscl $= 1$ and Yscl $= 1$ means that there is 1 unit between tick marks on each of the axes. In this text, scaling factors other than 1 will be listed by the window unless they are readily apparent.

After entering the equation $y = (3/5)x + 2$ and choosing a viewing window, we can then draw the graph.



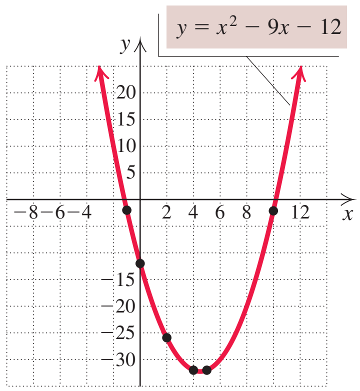
EXAMPLE 5 Graph: $y = x^2 - 9x - 12$.

Just in Time 7

Solution Note that since this equation is not of the form $Ax + By = C$, its graph is not a straight line. We make a table of values, plot enough points to obtain an idea of the shape of the curve, and connect the points with a smooth curve. It is important to scale the axes to include most of the ordered pairs listed in the table. Here it is appropriate to use a larger scale on the y-axis than on the x-axis.

x	y	(x, y)
−3	24	(−3, 24)
−1	−2	(−1, −2)
0	−12	(0, −12)
2	−26	(2, −26)
4	−32	(4, −32)
5	−32	(5, −32)
10	−2	(10, −2)
12	24	(12, 24)

① **Select values for *x*.**
② **Compute values for *y*.**



Now Try Exercise 39.

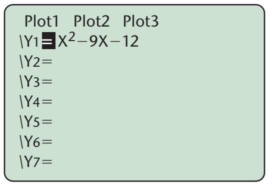


FIGURE 1.

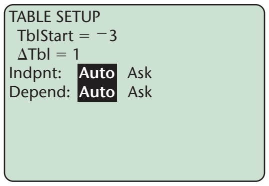


FIGURE 2.

A graphing calculator can be used to create a table of ordered pairs that are solutions of an equation. For the equation in Example 5, $y = x^2 - 9x - 12$, we first enter the equation on the equation-editor screen. (See Fig. 1.) Then we set up a table in AUTO mode by designating a value for TBLSTART and a value for ΔTBL. The calculator will produce a table beginning with the value of TBLSTART and continuing by adding ΔTBL to supply succeeding x-values. For the equation $y = x^2 - 9x - 12$, we let TBLSTART $= -3$ and ΔTBL $= 1$. (See Fig. 2.)

We can scroll up and down in the table to find values other than those shown in Fig. 3. We can also graph this equation on the graphing calculator, as shown in Fig. 4.

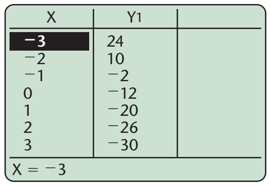


FIGURE 3.

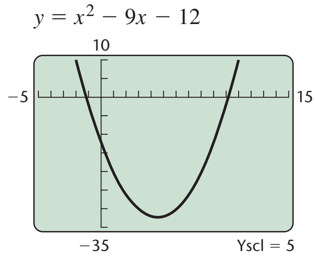
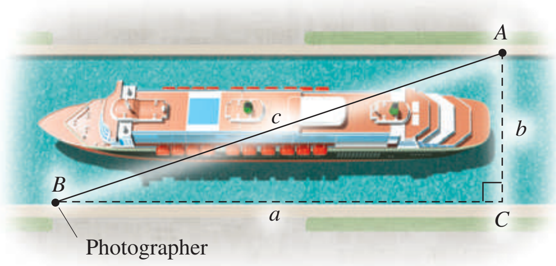


FIGURE 4.

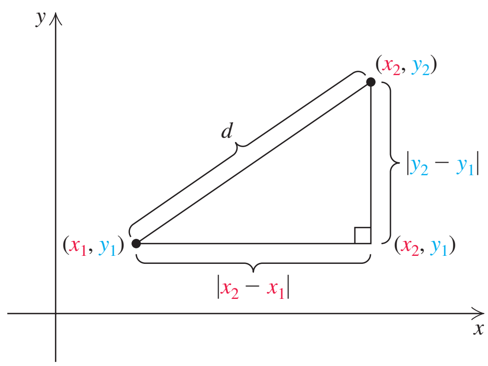
The \$5.25 billion expansion of the Panama Canal doubled its capacity. A third canal lane is scheduled to open in 2016. (Source: *Panama Canal Authority*)

The Distance Formula

Suppose that a photographer must determine the distance between two points, A and B, on opposite sides of a lane of the Panama Canal. One way in which he or she might proceed is to measure two legs of a right triangle that is situated as shown below. The Pythagorean equation, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the lengths of the legs, can then be used to find the length of the hypotenuse, which is the distance from A to B.



A similar strategy is used to find the distance between two points in a plane. For two points (x_1, y_1) and (x_2, y_2), we can draw a right triangle in which the legs have lengths $|x_2 - x_1|$ and $|y_2 - y_1|$.



Just in Time 4, 28

Using the Pythagorean equation, $c^2 = a^2 + b^2$, we have

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2.$$ **Substituting *d* for *c*, $|x_2 - x_1|$ for *a*, and $|y_2 - y_1|$ for *b* in the Pythagorean equation**

Because we are squaring, we can use parentheses to replace the absolute-value symbols:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

Taking the principal square root, we obtain the distance formula.

THE DISTANCE FORMULA

The **distance *d*** between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The subtraction of the *x*-coordinates can be done in any order, as can the subtraction of the *y*-coordinates. Although we derived the distance formula by considering two points not on a horizontal line or a vertical line, the distance formula holds for *any* two points.

Just in Time 5, 25

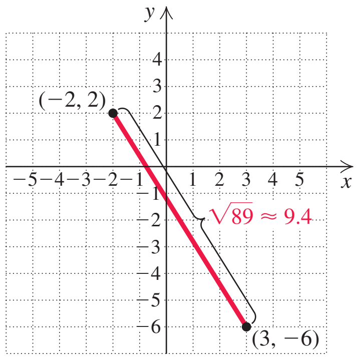
EXAMPLE 6 Find the distance between each pair of points.

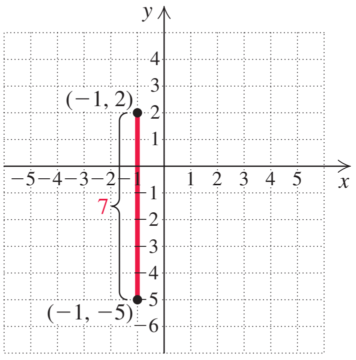
a) $(-2, 2)$ and $(3, -6)$ **b)** $(-1, -5)$ and $(-1, 2)$

Solution We substitute into the distance formula.

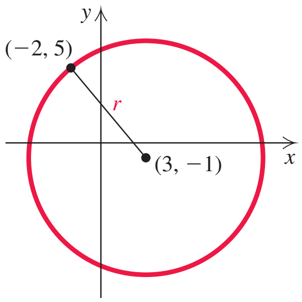
a)
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[3 - (-2)]^2 + (-6 - 2)^2} \\ &= \sqrt{5^2 + (-8)^2} = \sqrt{25 + 64} \\ &= \sqrt{89} \approx 9.4 \end{aligned}$$

b)
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-1 - (-1)]^2 + (-5 - 2)^2} \\ &= \sqrt{0^2 + (-7)^2} = \sqrt{0 + 49} \\ &= \sqrt{49} = 7 \end{aligned}$$





Now Try Exercises 63 and 71.



EXAMPLE 7 The point $(-2, 5)$ is on a circle that has $(3, -1)$ as its center. Find the length of the radius of the circle.

Solution Since the length of the radius is the distance from the center to a point on the circle, we substitute into the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2};$$

$$r = \sqrt{[3 - (-2)]^2 + (-1 - 5)^2}$$ **Substituting r for d, $(3, -1)$ for (x_2, y_2), and $(-2, 5)$ for (x_1, y_1). Either point can serve as (x_1, y_1).**

$$= \sqrt{5^2 + (-6)^2} = \sqrt{25 + 36}$$

$$= \sqrt{61} \approx 7.8.$$ **Rounding to the nearest tenth**

The radius of the circle is approximately 7.8.

Now Try Exercise 77.

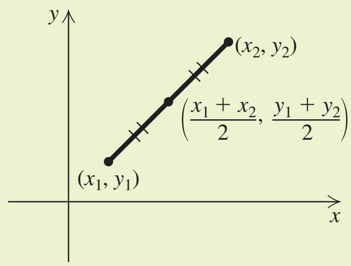
Midpoints of Segments

The distance formula can be used to develop a method of determining the *midpoint* of a segment when the endpoints are known. We state the formula and leave its proof to the exercises.

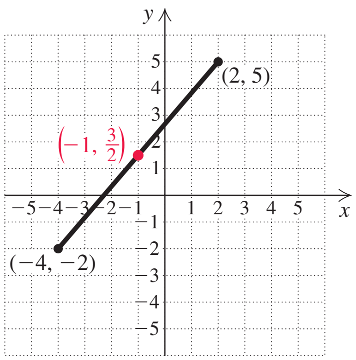
THE MIDPOINT FORMULA

If the endpoints of a segment are (x_1, y_1) and (x_2, y_2), then the coordinates of the **midpoint** of the segment are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



Note that we obtain the coordinates of the midpoint by averaging the coordinates of the endpoints. This is a good way to remember the midpoint formula.



EXAMPLE 8 Find the midpoint of the segment whose endpoints are $(-4, -2)$ and $(2, 5)$.

Solution Using the midpoint formula, we obtain

$$\left(\frac{-4 + 2}{2}, \frac{-2 + 5}{2}\right) = \left(\frac{-2}{2}, \frac{3}{2}\right) = \left(-1, \frac{3}{2}\right).$$

Now Try Exercise 83.